ECE 315 – MIDTERM

WINTER 1998 **A.P. FELZER**

You may consult **your** notes and any books you may have or borrow from the library as well as any computer software or plotting calculators to do the following problems. But you **may not** under any circumstances for any reason talk to any person about the exam except for Felzer. If you **do discuss** this exam or **in any way** make use of the work of others, you will **fail** the course and have a letter put in your file explaining why.

To get a good grade in this exam you must not only get the right answers but also make sure that your solutions are neat, complete, concise, make obvious what each problem is, make obvious how you're solving the problem and make obvious what your answer is. You also need to include and label all pertinent drawings, graphs and tables and equations.

Note that it is better to do a problem with brute force than not at all. But it's better to do a problem "simply". Include any pertinent computer printouts. Be sure to start early enough so that you have time to think about and double check your work. And finally, be sure to read the problems very carefully. And to ask me if you have any question about what is given and what is being asked for

1. Write out the page of notes you would use for this midterm if it was closed book

2. What do we mean by the probability of an event A.

3. Given two events A and B with $P(A) = 0.4$ and $P(B) = 0.7$. What are the maximum and minimum values of $P(A \cap B)$

4. What is the minimum number of times a fair die must be tossed in order for the probability of getting at least one six is at least 0.85. Carry out the experiment a whole bunch of times and then compare with your predicted result

5. Suppose the owner of a pizza parlor flips a coin for each of 10 toppings to decide which ones to give you. You like eight of the toppings but dislike the other two.
   a. What's the probability that you get a pizza with a topping you don't like
   b. What's the average number of toppings you get

6. Ten high school students make their lunches for an outting but forget to put their names on them. So at lunch time the lunches are handed out at random.
   a. What's the probability that Joe gets the lunch he made
   b. What's the probability that at least one person gets his or her own lunch

7. In Galileo's time people thought that when three dice were rolled a sum of 9 and a sum of 10 had the same probability since each could be obtained 6 ways as follows

   9: 1+2+6, 1+3+5, 1+4+4, 2+2+5, 2+3+4, 3+3+3
   10: 1+3+6, 1+4+5, 2+4+4, 2+3+5, 2+4+4, 3+3+4

   Show that the probability of 10 is really larger - and explain why

8. Five pennies are sitting on a table. One is a trick coin with two heads but the other four are normal. You pick up a coin at random and flip it four times - getting heads each time. What is the probability you picked up the two headed penny

9. What are random variables. Illustrate with an example different from any one we've used in this class

10. Given two random variables X and Y
    a. What do we mean when we say X and Y are independent
    b. Come up with an example of two independent random variables X and Y. Explain how you can tell that they're independent

11. Find the conditional probabilities P(X|Y) if the discrete random variables X and Y correspond to the outcomes of flipping a fair coin three times as follows

$$X = \text{number of heads} \qquad Y = \begin{cases} 0 & \text{First flip a tail} \\ 1 & \text{Odd number of heads} \end{cases}$$

12. Suppose the random variable X takes on the values 1 and 3 and the random variable Y takes on the values 2 and 4 for a particular random experiment. Now suppose we do the experiment a whole bunch of times and find that x = 1 for 30 times and x = 3 for 170 times. Find possible values for

    N_{12} = Number of times X = 1, Y = 2 $\qquad$ N_{14} = Number of times X = 1, Y = 4
    N_{32} = Number of times X = 3, Y = 2 $\qquad$ N_{34} = Number of times X = 3, Y = 4

    a. If X and Y are independent. Explain how you got your results
    b. If X and Y are not independent. Explain how you got your results

13. Suppose we have a communication system that transmits 1's and 0's such that the probability that a transmitted 0 is received correctly is 0.97 and the probability that a transmitted 1 is received correctly is 0.9. The probability that a 0 is transmitted is 0.4.
    a. Find the probability that a 1 is received
    b. Find the probability that a 1 was transmitted if a 1 is received

14. Suppose we define the discrete random variables X and Y corresponding to the flipping of a fair coin three times as follows

$$X = \text{number of heads} \qquad Y = \begin{cases} 0 & \text{Even number of heads} \\ 1 & \text{Odd number of heads} \end{cases}$$

    a. Find and plot $f_X(x)$ and $F_X(x)$
    b. Find and plot $f_Y(y)$ and $F_Y(y)$
    c. Find $f_{XY}(x,y)$. Then verify that your values are consistent with your results in parts (a) and (b)
    d. Are X and Y independent. Justify
    e. Find E[X] and var[X]
    f. Find E[Y] and var[Y]
    g. Find E[XY]
    h. Find the covariance σ_{XY}. What does your value for σ_{XY} tell you about the relationship between X and Y
    i. Are your results in parts (d) and (h) consistent. Explain

j. Find the correlation coefficient ρ
k. Find $E[X+Y]$
l. Carry out the experiment a whole of times and then compare your experimental value of $E[X+Y]$ with the value you predicted in part (k)

15. Show that if random variables X and Y are related by $Y = aX + b$ with $a > 0$, then the correlation coefficient $\rho = 1$

16. Suppose that the number of errors made by a given computer system has a Poisson distribution with an average of one error every 1000 hours of operation.
    a. What's the probability that it will make 4 errors in 3000 hours of operation
    b. What's the probability that the 4 errors in part (a) will occur in the middle 1000 hours. Explain how you got your answer
    c. What's the probability that 2 of the errors in part (a) come during the first 500 hours and the other 2 come during the second 500 hours. Explain why your answer is different from part (b)

17. Suppose that a modem has a bit error rate of 10^{-4}. What is the probability of no more than one error in the transmission of 10^4 bits

18. Show that if X_1 and X_2 are independent and identically distributed – iid – then

$$\text{Var}[X_1 + X_2] = 2\text{Var}[X_1]$$

19. Given

$$f_X(x) = \begin{cases} Ke^{-0.1x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

    a. Find K so that $f_X(x)$ is a probability density function. Then sketch your $f_X(x)$
    b. Find and then sketch the corresponding cumulative probability function $F_X(x)$

20. Suppose a computer system has an average of three major crashes every 600 days. What's the probability it will have no major crash in the next 200 days

21. The liftetime X of a light bulb is a random variable with $P[X > t] = 1/(1+t)$ $t \geq 0$. Find the probability that at least one of three such light bulbs is still working at time $t = 5$