ECE 315 – FINAL

WINTER 1998 **A.P. FELZER**

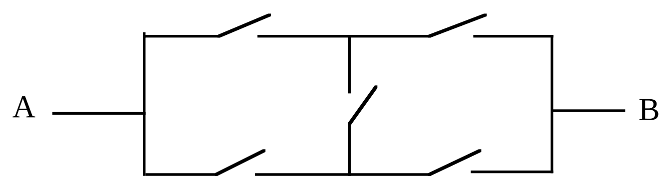
You may consult **your** notes and any books you may have or borrow from the library as well as any computer software or plotting calculators to do the following problems. But you **may not** under any circumstances for any reason talk to any person about the exam except for Felzer. If you **do discuss** this exam or **in any way** make use of the work of others, you will **fail** the course and have a letter put in your file explaining why.

To get a good grade in this exam you must not only get the right answers but also make sure that your solutions are neat, complete, concise, make obvious what each problem is, make obvious how you're solving the problem and make obvious what your answer is. You also need to include drawings of all circuits (including equivalent circuits) as well as appropriate graphs and tables. In addition all equations, graphs and tables must be labeled

Note that it is better to do a problem with brute force than not at all. But it's better to do a problem "simply". Include any pertinent computer printouts. Be sure to start early enough so that you have time to think about and double check your work

1. Write out the page of notes you would use for this final if it was closed book

2. What's the probability of there being a path from A to B in the following switching network



if each switch is equally likely to be open or closed and all the switches are independent

3. How many times do you need to toss a fair die before the probability of at least two sixes is at least 0.9

4. Two dice are thrown until the sum is seven. What is the average number of times the dice are thrown

5. Two kids take turns throwing darts at a target. A throws first and hits with probability of 1/4. B throws second and hits with probability 1/3. What is the probability that A will hit the target before B

6. Suppose for simplicity that the number of kids in a family is 1, 2 or 3 with probability 1/3 each. Little Bobby has no brothers. What is the probability he is an only child

7. Suppose you're in a room with three doors – one of which leads through a maze to the outside while the other two lead through mazes back to the room. Each maze takes three hours. Each time you return to the room you're in such a daze you don't know which door you entered or exited. What's the average length of time it takes to reach the outside

8. Suppose arriving phone calls have a Poisson distribution with $\lambda = 10$ calls/minute.
   a. What's the probability of 10 calls in the first minute and 10 calls in the last minute of a three minute interval if there are a total of 25 calls

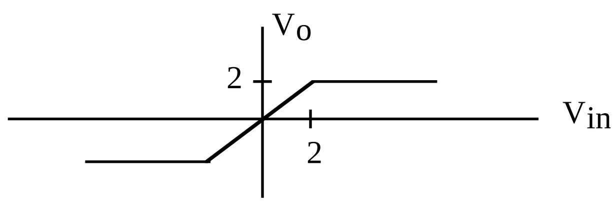
b. What's the probability of there being less than 5 seconds between two successive calls

9. It has been estimated that the spatial distribution of boulders in a given region of the Martian surface, which may be hazardous for landing a spacecraft, has a Poisson distribution with an average of $\lambda = 1000$ boulders per square kilometer
   a. Find the probability that there is no such boulder within a radius of 25 meters of a selected touch down point in the region
   b. Find the radius of the circle such that the probability of no boulder within it is equal to 0.5

10. Suppose you're in a long line of customers at Fry's waiting for the next available checker. And that people leaving the line going to a checker satisfy a Poisson distribution with $\lambda = 2$ persons per minute. What is the probability you will be in line at least 6 minutes if you are 10th in line

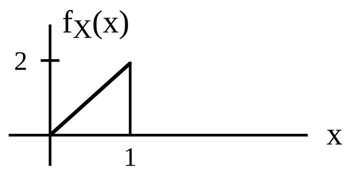
11. What would σ have to be so that 99.8% of the resistors coming off a production line were within 1% of 1K if the distribution is Gaussian with $\mu = 1000$

12. Suppose a Gaussian signal with $\mu = 0$ and $\sigma = 1.5$ volt is the input to a limiter with transfer characteristic as follows



What is the probability that $V_o = 2$ volts.

13. Given the following probability density function for the random variable X



find $f_Y(y)$ if $Y = 2 - X$

14 Find the probability density function $f_Y(y)$ of $y = 2e^{-x}$ if the random variable X is uniformly distributed in the range [0, 2]

15. Given the following joint probability density function

$$f_{XY}(x, y) = k(x + y) \qquad 0 \le x \le 1 \text{ and } 0 \le y \le 1$$

   a. Find k
   b. Find the joint cumulative probability function $F_{XY}(x, y)$
   c. Find the marginal probability density functions
   d. Are X and Y independent

16. Given the following joint density function

$$f_{XY}(x,y) = Ae^{-|x|-2|y|}$$

a. Find A. Be careful of the absolute value signs
b. Determine if X and Y are independent
c. Find f(X|Y)
d. Are your results in parts (b) and (c) consistent. Explain how you know

17. Suppose the continuous random variable X has a uniform probability density with mean μ and variance σ^2

a. What is the mean and variance of

$$Y = \frac{1}{5}\left(X_1 + X_2 + X_3 + X_4 + X_5\right)$$

where every value of Y is the average of five outcomes of X

b. Would you use a single value of X or a single value of Y to estimate E[X] – or does it matter which you use. Explain

18. Determine if the random process X(t) = A sin (ωt) is wide sense stationary if A is uniformly distributed over the range [0, 5]

19. Find the autocorrelation of a random process with sample functions of the form

$$X(t) = A \sin(\omega t + \theta)$$

if A and θ are independent random variables with A uniformly distributed over the range [0, 5] and θ is uniformly distributed over the range [0, π]

20. A sample function from an ergodic random process is sampled at 10 widely separated times with the following results

{6, 4, 8, 1, 1, 5, 4, 6, 5, 7}

a. Estimate the mean
b. Estimate the variance

21. Find the expectation of the square of a stationary random process with power spectral density as follows