ECE 409 - BASEBAND TRANSMISSION - INVESTIGATION 5
INTRODUCTION TO INTERSYMBOL INTERFERENCE

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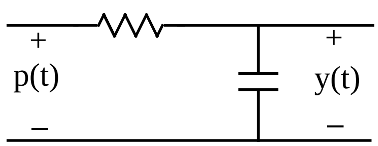
To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

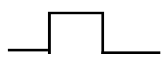
In the last Investigation we showed how to calculate the power spectral densities of baseband signals. The objective of this Investigation is to see how communication channels of limited bandwidth as follows

distort baseband signals and therefore affect the ability of the receiver to determine which signals are 1's and which are 0's.

1. We begin by analyzing the response of a channel that can be modeled by a simple first order lowpass circuit as follows

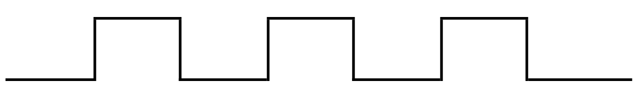


to a single pulse $p(t)$ as follows



a. Sketch the pulse response $y(t)$ of the circuit if the width of the pulse is 2τ
b. Describe the distortion - how $y(t)$ looks different from $p(t)$
c. Make use of time domain arguments - arguments that take into account the charging and discharging of the capacitor - to explain the distortion
d. Make use of frequency domain analysis - Fourier analysis - to explain the distortion

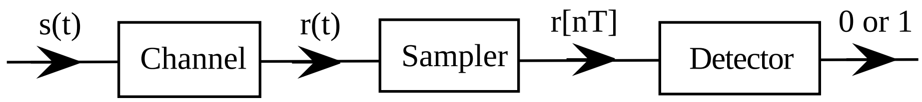
2. From Problem (1) we know that each pulse has a transient response. So if we now transmit three pulses like the following



over a communication channel then the still present transient response of the first pulse will interfere with the channel's response to the second and third pulses and so on. We call the resulting distortion **intersymbol interference (ISI)**. Assuming a pulse width of 3τ

a. Sketch the received signal if the time between pulses is equal to 5τ
b. Sketch the received signal if the time between pulses is equal to τ
c. Describe the differences between the responses
d. In which case is intersymbol interference the most
e. Would you expect ISI to be a problem in fiber optic systems. Why

3. From Problems (1) and (2) it looks like intersymbol interference is inevitable when bandwidths are limited. But in fact ISI can be greatly reduced in receivers that use center point detection as follows

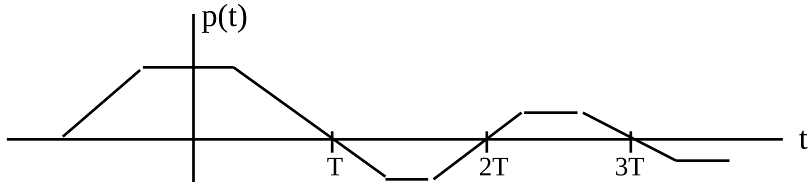


to *decide* which of the received signals are $1's$ and which are $0's$ as follows

(1) First the received signal $r(t)$ is sampled every T seconds

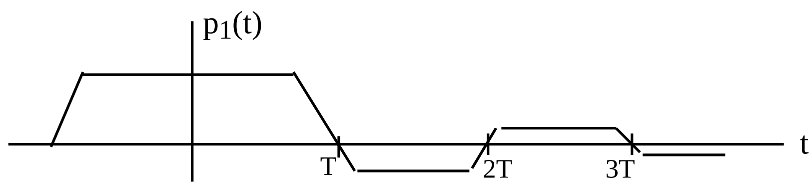
(2) Then a detector (just a comparator) decides based on the value of $r[nT]$ whether the corresponding bit is a 0 or a 1

The affects of ISI can then be minimized by using pulses that are zero at times T, 2T, 3T . . . as follows

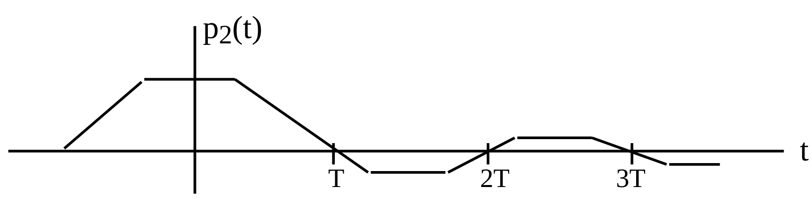


when subsequent pulses are being sampled for detection.

a. Why is the method just described referred to as *center point detection*
b. Sketch a Nyquist pulse for T = 0.1 msec
c. Explain in words how Nyquist pulses minimize the affects of intersymbol interference
d. Which of the following two pulses would do a better job of minimizing intersymbol interference in the real world



or



Hint - there will always be some jitter - there will always be some variations in the times the pulses are detected as a result of variations in the clocks.

4. From Problem (3) we know that we can reduce intersymbol interference if we use pulses that are zero every T seconds as follows

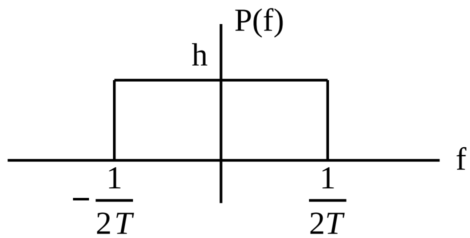
$$p(t) = \begin{cases} 1 & t = 0 \\ 0 & t = nT \end{cases}$$

when subsequent pulses are being detected. So the big question is - can we find pulses p(t) that satisfy this criteria that are also bandlimited so they won't be distorted by the finite bandwidth of the communication channel.

The "simplest" such pulses - which we refer to as **Nyquist pulses** - are sinc pulses as follows

$$p(t) = \frac{h}{T}\,\mathrm{sinc}\left(\frac{t}{T}\right)$$

with spectrums of bandwidth $BW = \frac{1}{2T} = \frac{1}{2} f_b$ as follows



a. Sketch $p(t)$
b. Verify that $p(t)$ is a Nyquist pulse

5. We now generalize on our results from Problem (4). Suppose we wish to transmit data at the rate of f_b bits/sec with **Nyquist pulses** - pulses of finite bandwidth like the one in Problem (4) that do not have intersymbol interference at the sampling times nT. Then it can be shown that

   (1) Nyquist pulses must have bandwidths of at least $f_b/2$

   (2) If a pulse $p(t)$ with a bandwidth BW in the range

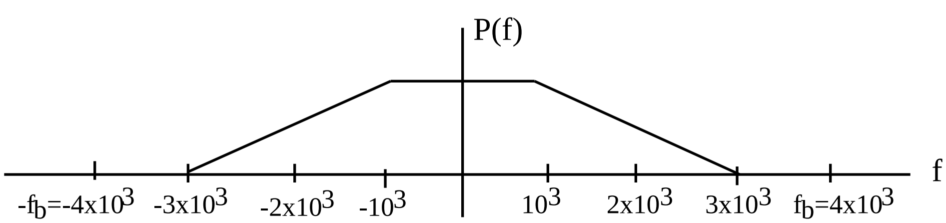
   $$\frac{f_b}{2} \le BW \le f_b \qquad T = \frac{1}{f_b}$$

   satisfies the condition

   $$P(f + f_b) + P(f) + P(f - f_b) = K \qquad -f_b \le f \le f_b$$

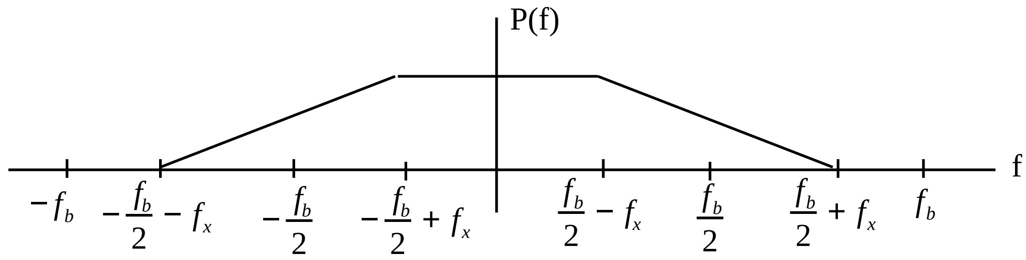
   then it will be a Nyquist pulse. Note that all Nyquist pulses that meet these conditions are said to have Nyquist frequency spectrums

   a. Verify that the following spectrum satisfies the conditions of a Nyquist spectrum



   b. Come up with your own example of a Nyquist spectrum

6. If we now draw Nyquist spectrums P(f) as follows



for pulses p(t) being transmitted at the rate

$$f_b = \frac{1}{T} \quad \text{bits/sec}$$

then the bandwith B_T needed to transmit these pulses without intersymbol interference is equal to

$$B_T = \frac{f_b}{2} + f_x$$

If we now define the **roll-off factor r** as follows

$$r = \frac{f_x}{f_b/2} \qquad \text{then} \qquad B_T = (1+r)\frac{f_b}{2}$$

a. Does a small r mean a slow or fast rolloff of the frequency response
b. Find the bandwidth B_T required for transmitting pulses at a rate of $1.2\text{x}10^4$ bits/sec with a roll-off factor of $r = 0.7$
c. What's the highest bit rate for pulses being transmitted over a channel with bandwidth $B_T = 20\text{KHz}$ for a roll-off factor of $r = 0.6$

7. A class of Nyquist pulses called **raised-cosine** pulses as follows

$$P(f) = \begin{cases} 1 & \text{when } |f| < \frac{f_b}{2} - f_x \\ \frac{1}{2}\left(1 - \sin\left[\frac{\pi\left(f - \frac{f_b}{2}\right)}{2 f_x}\right]\right) & \text{when } \left|f - \frac{f_b}{2}\right| \le f_x \\ 0 & \text{when } |f| > \frac{f_b}{2} + f_x \end{cases}$$

are particularly popular because of their relatively smooth frequency responses

a. Use Mathcad, Matlab or an equivalent to obtain plots of P(f) for roll-offs of $r = 0,\ 0.5$ and 1. Hint - choose a value for f_b and then calculate f_x from r
b. Describe how the plots of $P(f)$ change as the roll-off r increases
c. Verify that when $r = 1$ and $f_x = f_b/2$ then $P(f)$ becomes

$$P(f) = \frac{1}{2}\left(1 + \cos\left(\frac{\pi f}{f_b}\right)\right) \qquad \text{for } |f| \le f_b$$

d. Plot the raised cosine pulse $p(t)$ for $r = 1$ as follows

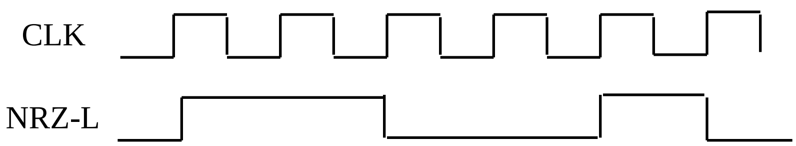
$$p(t) = f_b \frac{\cos(\pi f_b t)}{1 - 4 f_b^2 t^2} \operatorname{sinc}(f_b t)$$

obtained by taking the inverse Fourier Transform $P^{-1}(f)$. Mark the times T, 2T and 3T

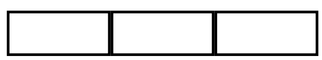
e. Describe what $p(t)$ looks like in part (d). Do you think it will do a good job of minimizing intersymbol interference. How can you tell

8. Describe how a lookup table can be used to generate raised-cosine pulses

9. The objective of this problem is to introduce **eye patterns** for displaying distortion caused by intersymbol interference. If we display an ideal NRZ-L signal as follows

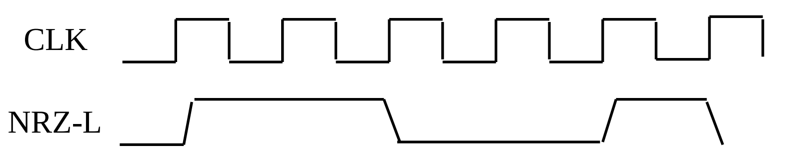


on a scope externally triggered by the rising edge of the clock pulses then we'll see a nice "clean" rectangular pattern as follows

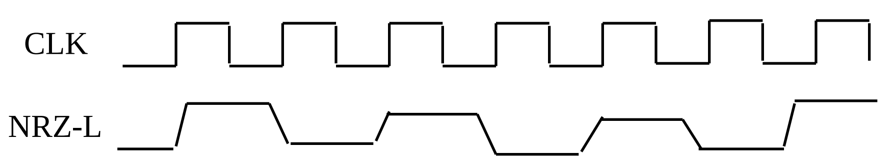


We call such a signal an eye pattern because as the NRZ-L becomes distorted the signal displayed on the scope will look more and more like an eye

a. Sketch the eye pattern of the following NRZ-L signal with finite rise and fall times



b. Sketch the eye pattern of the following NRZ-L signal as follows with not only finite rise and fall times but also distorted amplitudes



c. Sketch the eye pattern of a received NRZ-L signal with a lot of distortion

d. Make use of your results in parts (a)-(c) to describe the difference between eye patterns when there is little distortion in contrast to when there is a lot of distortion.