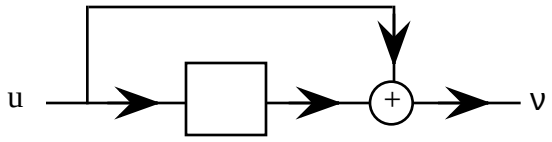
ECE 409 - ERROR CORRECTING CODES - INVESTIGATION 24
INTRODUCTION TO CONVOLUTION CODES - PART I

SUMMER 2004 **A.P. FELZER**

To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

Up to now we've been working with block codes of size (n,k) with n bit codewords that contain k bits of data. Such codes are characterized by the fact that any given bit of data affects only its block of output. The objective of this Investigation is to introduce convolution codes of size (n,k,m) with n-bit outputs that depend not only on the k bits of input in the block but also on m bits of previous inputs.

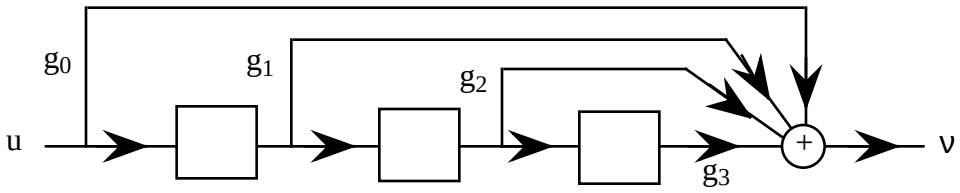
1. **Convolution codes** of size (n,k,m) are codes with n-bit outputs that depend on k bits of present input and m bits of previous input. The objective of this problem is to introduce convolution codes with the following simple circuit



with input $u = u_0 u_1 u_2 \cdots$ and output $\nu = \nu_0 \nu_1 \nu_2 \cdots$ that transmits data in blocks of n=1 with k=1. Note that the box contains a D flip-flop and as usual the addition is mod (2)

   a. What makes this circuit a convolution encoder
   b. What is m for this circuit
   c. Find the output ν for the input u = 1001101. Assume that the first bit entered is the bit on the left and that the flip-flop is initially cleared
   d. Find the input data u if ν = 10110 if the first bit received is the bit on the left

2. Now suppose we add two more flip-flops to our circuit in Problem (1) as follows

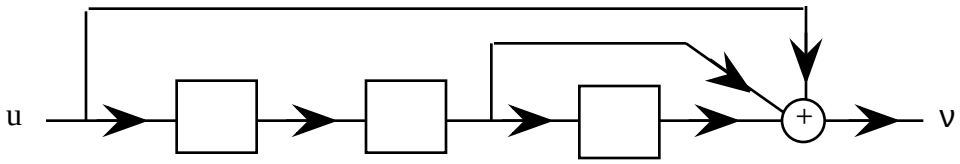


We call this a **linear feed-forward shift register**. The values of the g_k's are 0 or 1 depending on whether or not the link is present. Assuming $g_0 = g_1 = g_2 = g_3 = 1$

   a. How many output bits does any given input bit affect
   b. What is the size (n,k,m) of this circuit
   c. Find the response to u=100101. Continue inputting 0's until the last bit of u has worked it's way through the circuit. Again assume that the first input bit is the one on the left.
   d. Write out a general e**x**pression for ν_j in terms of the u_j's and g_k's
   e. Make use of your result in part (d) to verify that ν is equal to the convolution of u and g as follows

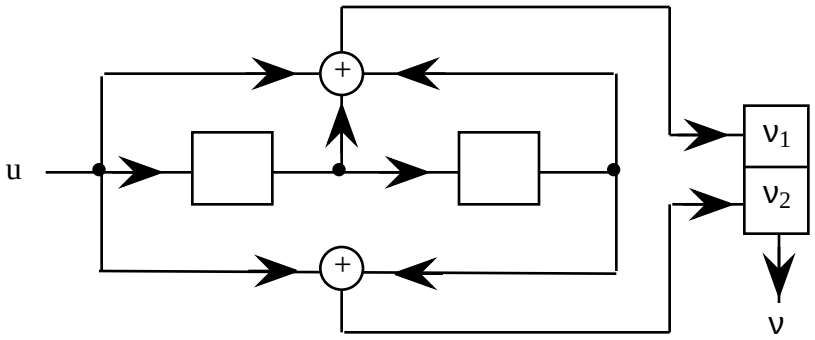
$$\nu = u \quad g$$

3. Given the following convolution encoder



   a. Write out equations for the first 4 outputs ν_0 to ν_3 in terms of the inputs u_0 u_1 . . .
   b. Redraw the circuit with a z^{-1} in each box to put it in the form of a digital filter
   c. Is your circuit in part (b) an FIR or IIR filter. How can you tell

4. Given the following convolutional encoder



   that generates two bit outputs $\nu_1\nu_2$ for every input bit
   a. What is the size (n,k,m) of this convolution code
   b. How many output bits are influenced by any given bit
   c. Find the output ν for the input u=100101
   d. Why is the output 00 00 00 10 not possible
   e. Redraw the circuit as a digital filter. Use mod(2) adders that have only two inputs

5. Use circuits like those in Problem (3) together with an output register like in Problem (4) to realize a (2,1,3) convolution encoder. Assume all g_k' $s = 1$