ECE 409 - INTROD TO DIGITAL COMMUNICATIONS - INV 2
INTRODUCTION TO DIGITAL COMMUNICATIONS - PART II

SUMMER 2004 **A.P. FELZER**

To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

In the last Investigation we reviewed sampling and the conversion of the samples to signed binary. These digital signals have a number of advantages including the following

(1) Digital signals are more immune to noise

(2) Digital signals from different sources ranging from computers to digitized voice and video can all be handled in the same way by digital communication systems

(3) Audio and video digital signals can be *compressed* to reduce the data rate

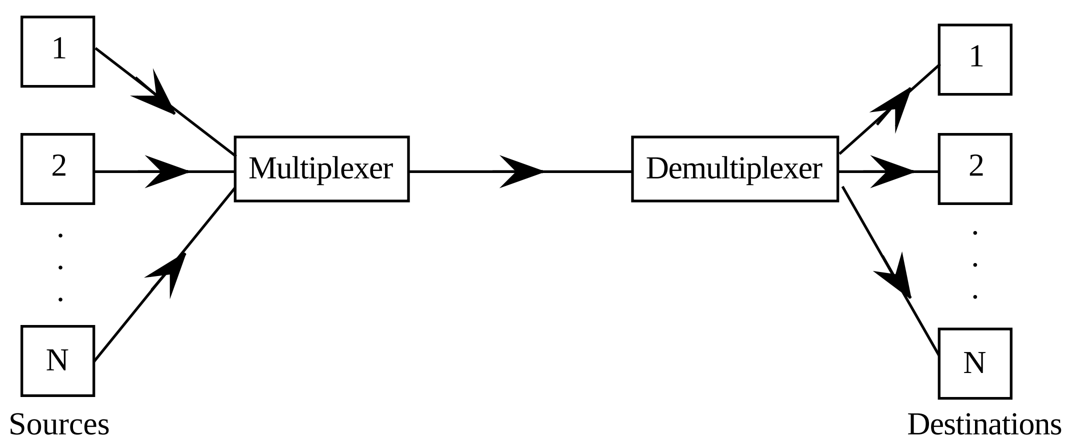
The objectives of this Investigation are to show

(1) How time division multiplexing enables digital signals from aribtary sources to share digital communication channels

(2) How companding reduces the data rate of telephone signals.

1. In analog communication systems like those for AM, FM and TV we make use of frequency division multiplexing (FDM) to transmit more than one signal at a time. Explain how FDM works

2. From Problem (1) we know that FDM works well in many applications. But the amount of filtering that would be required for the transmission of millions of phone calls, not to mention music, graphics and video gets pretty large pretty fast. The alternative for digital signals is to use **time division multiplexing (TDM)** where the digital signals from different sources share time on a communication channel as follows



What's the role of the multiplexer and demultiplexer in such a system

3. Suppose we want to build a digital system to time division multiplex the following three signals

(1) $m_1(t) =$ 5 KHz signal

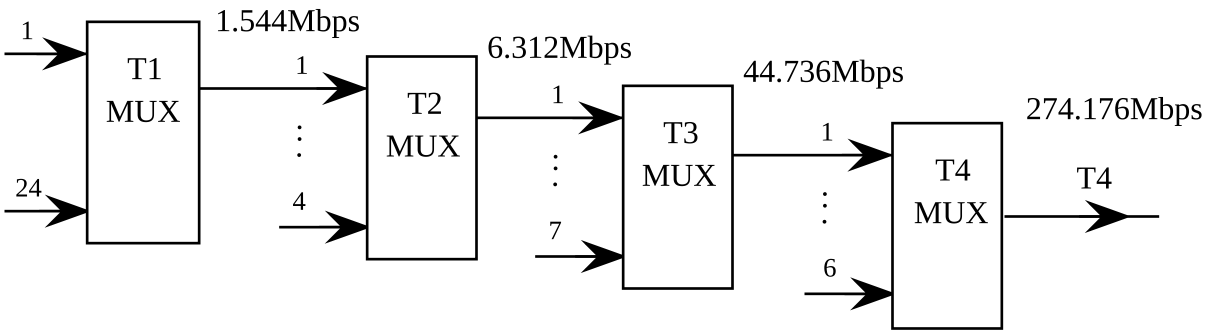
(2) $m_2(t) =$ 10 KHz signal

(3) $m_3(t) =$ = 5 KHz signal

Assume the signals are sampled at their Nyquist rate and then converted to 8 bits

a. What is the overall bit rate = the total number of bits/sec
b. How long does one complete cycle take - the time for $m_1(t)$ and $m_2(t)$ to each send 8 bit samples and $m_3(t)$ to send two 8 bit samples

4. Time division multiplexing systems are usually organized in hierarchies with individual signals first combined in small groups, then the small groups multiplexed together to form larger groups and so on. In the telephone system put together by AT&T as follows



the hierarchy is designed specifically to accomodate speech signals sampled at 8000 samples/secs with 8 bits per sample. How many telephone conversations can be serviced simultaneously by a T4 line

5. The objective of this and the rest of the problems in this Investigation is to introduce **companding** - a method used to compress the numbers of bits in digital telephone to 8 bits/sample. Generalizing on our roundoff error result in the last Investigation we know that when the roundoff error q of an analog-to-digital converter (ADC) with resolution Δ is uniformly distributed then the corresponding **roundoff noise** has an average power as follows

$$\text{Average Power of Roundoff Error } = E\left[Q^2\right] = \frac{\Delta^2}{12}$$

Make use of this result to find the average power of the roundoff error for a bipolar ADC with

$$\Delta = \frac{2\text{m}_{\text{max}}}{2^n} = \frac{2\text{m}_{\text{max}}}{\text{Number of Quantization Levels}}$$

when $m_{\max} = 5$ and n=8

6. Clearly we want the noise created by the roundoff errors of our ADCs to be as small as possible. But that by itself is not enough. What's really important - as in the case of analog signals - is the signal-to-noise ratio $(\text{SNR})_O$ at the outputs of our ADCs as follows

$$(SNR)_o = \frac{\text{Average Power of the Signal m(t)}}{\text{Average Power of the Noise}}$$

Suppose in particular that m(t) is equal to the following sinusoid

$$m(t) = m_p \cos(2\pi f t)$$

a. Find the average power of m(t)
b. Make use of your result in part (a) to show that the $(SNR)_o$ in dB is given by

$$10\log_{10}(SNR)_o = 6n + 1.76 \text{ dB}$$

7. How does the $(SNR)_o$ of $m_1(t)$ compare with that for $m_2(t)$ if most of the time $m_1(t)$ is small with $|m_1(t)| < 0.5m_{max}$ in contrast to $m_2(t)$ which is mostly large with $|m_2(t)| > 0.5m_{max}$

8. From Problem (7) we see that the smaller the average power of m(t) the smaller the $(SNR)_o$ for a given number of bits n. So this presents a dilemna - at least for voice transmission over the telephone. Since most speech is relatively small in amplitude we need at least 30dB signal-to-noise ratio for the lowest voice signals in addition to an additional 40dB of dynamic range to accomodate the range of speakers. But this all adds up to 70dB or 12 bits - which is 4 more bits than we want to use.

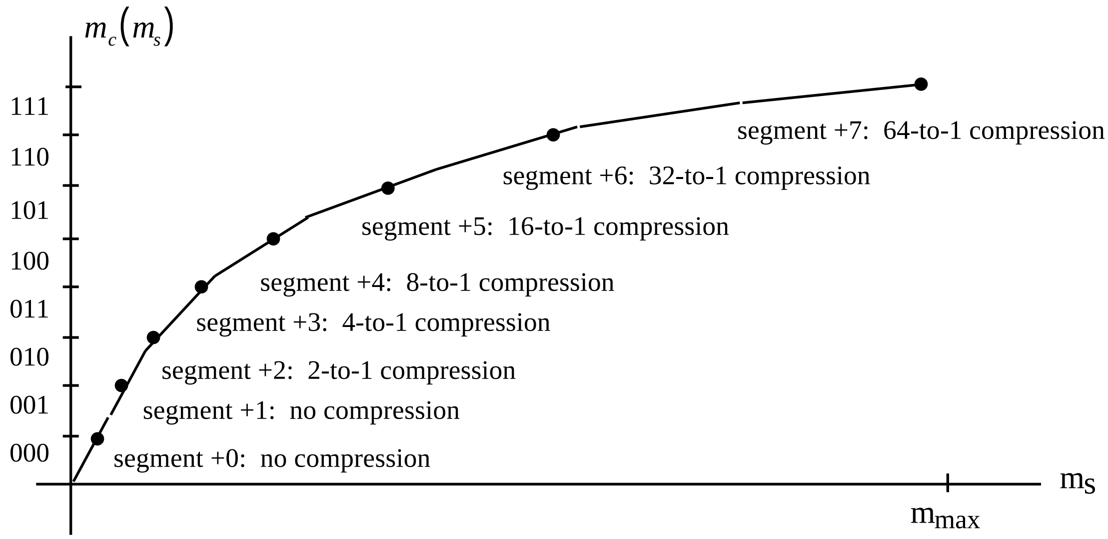
   The way we get around this problem is to simply transmit voice signals with a **compressed** PCM code that has lower resolution for large sample values and therefore requires fewer bits. This reduces the overall $(SNR)_o$ but not by much since voice communication contains relatively few large amplitude signals. At the receiver the compressed signal is **expanded** back to the larger number of bits. The whole process is referred to as **companding**. Chips that do this as well as other "chores" are referred to as **codecs** (coders/decoders). The compression standard used in the United States, Canada and Japan is called the µ-law as follows

$$m_c(m_s) = \text{sgn}(m_s) m_{\max} \frac{\ln\left(1 + \mu \left|\frac{m_s}{m_{\max}}\right|\right)}{\ln(1+\mu)}$$

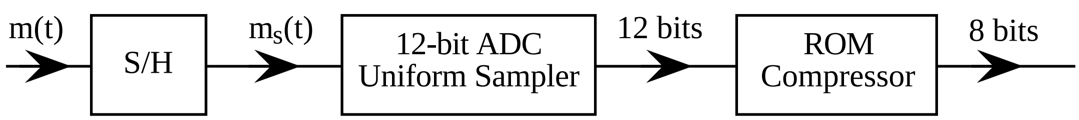
   where $\text{sgn}(m_s) = \text{sign of } m_s$. Note that Europe uses a similar A-law characteristic.

   a. Convert $m_s = 0.65$ to 12-bit signed binary with $m_{\max} = 10$
   b. Now calculate $m_c(m_s)$ for $m_s = 0.65$ with $m_{\max} = 10$. And then convert your $m_c(m_s)$ to 8-bit signed binary
   c. Compare your results in parts (a) and (b)
   d. Repeat parts (a)-(c) for $m_s = 8.35$

9. A common way to implement µ-law compression with $\mu = 255$ is to first *approximate* the µ-law curve with a series of eight straight line segments as shown in the following graph for positive sample values m_s. Note in particular that each segment is uniformly divided into $2^4 = 16$ codes



And then implement the 12-bit to 8-bit compression with a ROM lookup table as follows



a. Complete the following table

Segment	12 bits From ADC	Compressed PCM
+0	s0000000ABCD	0000ABCD
+1	s0000001ABCD	0001ABCD
+2	s000001ABCDx	0010ABCD
+3	s00001ABCDxx	0011ABCD
.	.	.
.	.	.

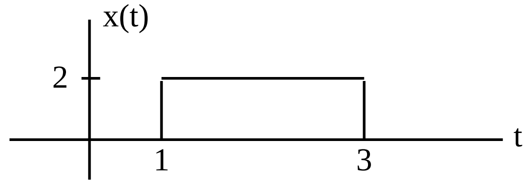
with a row for each segment specifying the relationship between the 12-bit signed binary output of the ADC and the 8-bit compressed code at the output of the ROM with

MSB = s = sign of the sample (0 for +; 1 for –)
Next 3 bits = segment number = location of most significant 1
Last 4 bits = ABCD = value in the segment = 4 bits after the first 1

b. Make use of your Table in part (a) to find the $\mu = 255$ compressed PCM code for 0011 0111 0101
c. Find the $\mu = 255$ compressed PCM code for $m_s = 0.65$ with $m_{\max} = 10$ by first converting m_S to 12-bit signed binary and then making use of your Table in part (a) to find the compressed PCM signal
d. Compare your compressed signals in part (c) and Problem (9)

10. Fourier Review - Find and sketch the Fourier Transforms of
   a. $x(t) = 10\cos(2\pi 1000t) + 5\cos(2\pi 2000t)$
   b. $x(t) = 5\delta(t)$

11. Math Review - Given the following signal



a. Sketch $x(t+1)$
b. Sketch $x(t)x(t+1)$

12. Make use of Mathcad or Matlab to obtain a graph of three periods of $x(t) = 5\cos(2\pi 1000t)$