ECE 409 - BANDPASS TRANSMISSION - INVESTIGATION 15
INTROD TO QUATERNARY PHASE SHIFT KEYING - PART II

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To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

In the last Investigation we introduced Quaternary Phase Shift Keying (QPSK) uses four sinusoids of the same amplitude and frequency but different phases as follows

$$s_i(t) = \sqrt{\frac{2E}{T}}\cos\left[2\pi f_c t + (2i-1)\frac{\pi}{4}\right] \qquad 0 \le t \le T$$

to transmit two bits of data. In particular we showed how QPSK signals can be expressed in terms of orthonormal basis functions. In this Investigation we show how orthonormal basis functions facilitate the generation and detection of QPSK signals.

1. Making use of our results in the last Investigation we see that if $x_1(t)$ and $x_2(t)$ are polar NRZ_L message signals of amplitude $\sqrt{E/2}$ then we can write

   $$s_i(t) = \sqrt{\frac{2E}{T}}\cos\left[2\pi f_c t + (2i-1)\frac{\pi}{4}\right] \qquad 0 \le t \le T$$

   as follows

   $$s_{QPSK}(t) = x_1(t)\phi_1(t) + x_2(t)\phi_2(t) = x_1(t)\sqrt{\frac{2}{T}}\cos(2\pi f_c t) + x_2(t)\sqrt{\frac{2}{T}}\sin(2\pi f_c t)$$

   in terms of the basis functions $\phi_1(t)$ and $\phi_2(t)$. Draw the block diagram for a circuit to generate $s_{QPSK}(t)$ as a sum of its basis functions

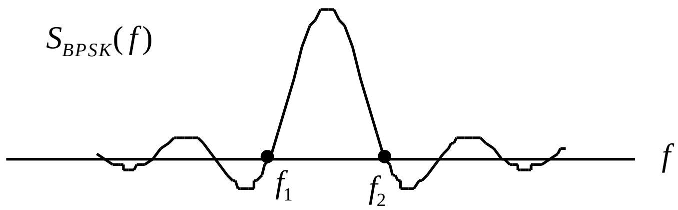
2. Draw a block diagram for a circuit to detect the two bits in a QPSK signal. Hint - obtain the first bit by multiplying the incoming QPSK signal by $\phi_1(t)$ and then take advantage of the fact that $\phi_1(t)$ and $\phi_2(t)$ are orthogonal.

3. Make use of the fact that the time average of the autocorrelation of QPSK is given by

   $$\langle R_X(t+\tau,t)\rangle = \frac{1}{T}R_{X_1}(\tau)\cos(2\pi f_c\tau) + \frac{1}{T}R_{X_2}(\tau)\cos(2\pi f_c\tau)$$

   to find its power spectral density $S_{QPSK}(f)$

4. Suppose we wish to transmit 10^6 bits/sec
   a. Find the null-to-null bandwidth for BPSK equal to $f_2 - f_1$ as follows



b. Find the null-to-null bandwidth for QPSK
c. Does BPSK or QPSK have a smaller bandwidth for a given bit rate. By how much

5. The objective of this problem is to calculate the probability of error P_e of QPSK signals as follows

$$s_{QPSK}(t) = x_1(t)\sqrt{\frac{2}{T}}\cos(2\pi f_c t) + x_2(t)\sqrt{\frac{2}{T}}\sin(2\pi f_c t)$$

with $x_1(t)$ and $x_2(t)$ equal to polar NRZ_L message signals of amplitude $\sqrt{E/2}$ as above. Note that each of the terms in this sum is a BPSK signal of amplitude $A_c = \sqrt{E/T}$ with probability of error

$$P(E_1) = P(E_2) = Q\left(\sqrt{\frac{A_c^2 T}{N_o}}\right)$$

a. Find $P(E_1) = P(E_2)$ as a function of E_b and N_o. Note that $E = 2E_b$
b. Now the probability of error P_e is the probability there's an error in the first bit, second bit or both. So

$$P_e = P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

But since the two bits are independent we have

$$P_e = P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1)P(E_2)$$

Make use of this result to find P_e as a function of E_b and N_o

c. Make use of your result in part (b) to justify the fact that a good approximation of P_e is

$$P_e \approx 2Q\left(\sqrt{\frac{2E_b}{N_o}}\right)$$

6. Use the results of this Investigation and the Investigation on BPSK to compare the
a. Power spectral densities of QPSK and BPSK
b. Average energies per bit E_b of QPSK and BPSK
c. Bit error rate of QPSK and BPSK

7. What is the advantage of QPSK over BPSK