ECE 409 - BANDPASS TRANSMISSION - INVESTIGATION 14
INTROD TO QUATERNARY PHASE SHIFT KEYING - PART I

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To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

Up to now all of our bandpass modulation schemes have been binary with one sinusoid for 1 and another with a different amplitude, frequency or phase for 0. The objective of this and the next Investigation is to introduce Quaternary Phase Shift Keying (QPSK) in which we use four sinusoids of the same amplitude and frequency but different phases. We will show, in particular, how to express QPSK signals in terms of orthonormal basis functions and how this facilitates their generation and detection.

1. As we said in the introduction **Quaternary Phase Shift Keying (QPSK)** uses four sinusoids of the same amplitude and frequency but with different phases. We refer to these sinusoids as **symbols**
   a. How many bits of information are contained in a QPSK symbol.
   b. How does the amount of information transmitted by a QPSK system in a given time compare with that of a binary system like BASK, BFSK and BPSK
   c. We refer to the rate at which symbols are being transmitted as the **baud rate**. Memorize this term and then find the bit rate of a QPSK signal with a baud rate of 1000 symbols/sec

2. Given that the the sinusoids of QPSK signals are as follows

$$s_i(t) = \sqrt{\frac{2E}{T}}\cos\left[2\pi f_c t + (2i-1)\frac{\pi}{4}\right] \qquad 0 \le t \le T$$

   for $i = 1,2,3,4$ where E is the energy and T the duration of $s_i(t)$
   a. Verify that E is the energy in T seconds
   b. Find the phases
   c. Write out the equations for $s_1(t)$, $s_2(t)$, $s_3(t)$ and $s_4(t)$. Make phases between $-\pi$ and $+\pi$. Put your results in a Table

3. Given the following assignment of phase with data

i	phase	data
1	$\pi/4$	1 0
2	$3\pi/4$	0 0
3	$-3\pi/4$	0 1
4	$-\pi/4$	1 1

   sketch the QPSK signal for 11 00 01

4. Draw the eye pattern for a QPSK signal assuming the scope is being triggered by the system clock with frequency $f_o = 1/T$

5. Show that QPSK signals as follows

$$s_i(t) = \sqrt{\frac{2E}{T}}\cos\left[2\pi f_c t + (2i-1)\frac{\pi}{4}\right] \qquad 0 \le t \le T$$

can be expressed in the form

$$s_i(t) = s_{i1}\sqrt{\frac{2}{T}}\cos(2\pi f_c t) + s_{i2}\sqrt{\frac{2}{T}}\sin(2\pi f_c t)$$

with $\quad s_{i1} = \sqrt{E}\cos\left[(2i-1)\frac{\pi}{4}\right] \quad$ and $\quad s_{i2} = \sqrt{E}\sin\left[-(2i-1)\frac{\pi}{4}\right]$

6. From Problem (5) we know that $s_i(t)$ can be expressed as follows

$$s_i(t) = \sqrt{\frac{2E}{T}}\cos\left[2\pi f_c t + (2i-1)\frac{\pi}{4}\right] = s_{i1}\phi_1(t) + s_{i2}\phi_2(t)$$

where $\phi_1(t) = \sqrt{\frac{2}{T}}\cos(2\pi f_c t)$ and $\phi_2(t) = \sqrt{\frac{2}{T}}\sin(2\pi f_c t)$. We call $\phi_1(t)$ and $\phi_2(t)$ **basis functions** because each $s_i(t)$ can be written as a linear combination of them. The objective of this problem is to show that $\phi_1(t)$ and $\phi_2(t)$ are **orthonormal basis functions** - that they're both **orthogonal** and **normal**. **Memorize** this definition and then

a. Show that $\phi_1(t)$ and $\phi_2(t)$ are orthogonal - that they satisfy

$$\int_0^T \phi_1(t)\phi_2(t)\,dt = 0$$

b. Show that $\phi_1(t)$ and $\phi_2(t)$ are normal - that they satisfy

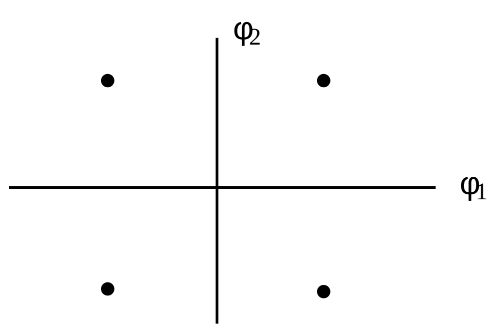
$$\int_0^T \phi_1^2(t)\,dt = 1 \quad \text{and} \quad \int_0^T \phi_2^2(t)\,dt = 1$$

c. Write out the signals $s_i(t)$ to transmit 11 00 01 in terms of the basis functions when $f_c = 10^8$ Hz

7. From Problems (5) and (6) we know that QPSK signals can be expressed in terms of orthonormal basis functions as follows

$$s_i(t) = s_{i1}\phi_1(t) + s_{i2}\phi_2(t)$$

And so we say they form a **two dimensional vector space** called a **signal space**. The objective of this problem is to introduce **constellation diagrams** as follows



for showing the "locations" of the members of QPSK signal spaces

a. Calculate the coordinates of the dots s_{i1} and s_{i2} for $i = 1,2,3,4$. Put your results in a Table
b. Now make use of your results in part (a) to locate $s_1(t)$, $s_2(t)$, $s_3(t)$ and $s_4(t)$ with coordinates (s_{i1}, s_{i2}) on the diagram