ECE 409 - BANDPASS TRANSMISSION - INVESTIGATION 13
INTRODUCTION TO BINARY PHASE SHIFT KEYING

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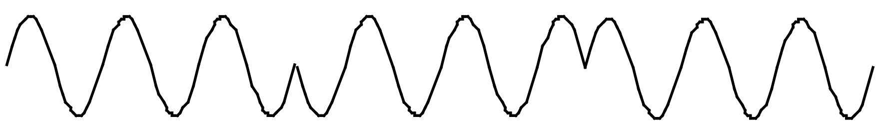
To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

In the last three Investigations we calculated the spectrums of BASK and BFSK signals, showed how they could be detected and calculated the probability of bit error. The objective of this Investigation is to do the same for Binary Phase Shift Keying (BPSK) signals.

1. **Binary Phase Shift Keying (BPSK)** makes use of sinusoids with two different phases as follows

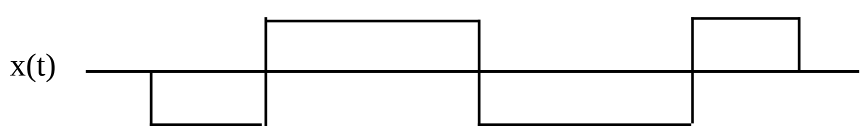
   $$s_1(t) = A_c\cos(2\pi f_c t) \quad \text{and} \quad s_2(t) = A_c\cos(2\pi f_c t + \pi) = -A_c\cos(2\pi f_c t)$$

   to transmit 1's and 0's as follows

   Draw a BPSK signal for 110100 using $s_1(t)$ for 1 and $s_2(t)$ for 0

2. Draw a circuit using a multiplier to generate the BPSK signal $x_{BFSK}(t)$ from NRZ-L polar message signals x(t) of magnitude 1 like the following

3. The objective of this problem is to find the power spectral density of BPSK signals as follows

   $$x_{BPSK}(t) = x(t)A_c\cos(2\pi f_c t)$$

   where $x(t)$ is a NRZ_L polar signal like in Problem (2)

   a. Show that $R_X(\tau) = \dfrac{1}{T}\sum_{m=-\infty}^{\infty} R_A(m)R_p(\tau - mT) = \text{tri}\left(\dfrac{\tau}{T}\right)$

   b. Show that $S_X(f) = T\,\text{sinc}^2(Tf)$

   c. Make use of the result in part (b) together with

   $$S_Y(f) = F\left[R_Y(\tau)\right] = F\left[\frac{A_c^2}{2}R_X(\tau)\cos(2\pi f_c\tau)\right] = \frac{A_c^2}{4}\left[S_X(f - f_c) + S_X(f + f_c)\right]$$

   to find the power spectral density $S_{BPSK}(f)$

   d. Make use of your result in part (c) to sketch the power spectral density $S_{BPSK}(f)$

4. Draw a block diagram of a coherent correlator BPSK detector. Describe how it differentiates between 1's and 0's

5. Show that the average energy E_b of a BPSK signal is equal to

$$E_b = \frac{A_c^2 T}{2}$$

6. Show that the probability of error P_e for BPSK is given by

$$P_e = Q\left(\sqrt{\frac{A_c^2 T}{N_o}}\right) = Q\left(\sqrt{\frac{2E_b}{N_o}}\right)$$

where $Q(a) = \frac{1}{\sqrt{2\pi}} \int_a^{\infty} e^{-z^2/2}\, dz$. Hint - be sure to calculate γ

7. Compare each of the following BASK, BFSK and BPSK
   a. Power spectral densities
   b. Average energy per bit E_b
   c. Bit error rate P_e

8. Would you build a bandpass digital communication system with BASK, BFSK or BPSK. Why