ECE 409 - BANDPASS TRANSMISSION - INVESTIGATION 12
INTRODUCTION TO BINARY FREQUENCY SHIFT KEYING

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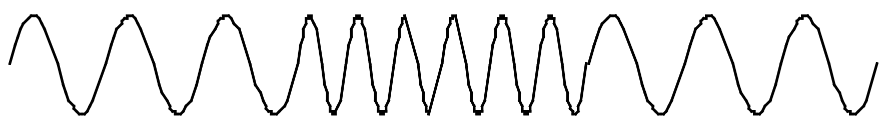
To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

In the last two Investigations we calculated the spectrums of BASK signals, showed how they could be detected and calculated the probability of bit error. The objective of this Investigation is to do the same for Binary Frequency Shift Keying (BFSK)

1. **Binary Frequency Shift Keying (BFSK)** makes use of sinusoids of two different frequencies as follows

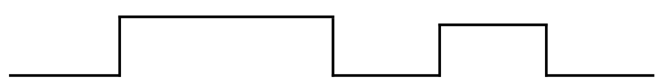
$$s_1(t) = A_c \cos\left(2\pi f_1 t\right) \quad \text{and} \quad s_2(t) = A_c \cos\left(2\pi f_2 t\right)$$

to transmit 1's and 0's as follows



Draw a BFSK signal for 110100 assuming 1 is the higher frequency sinusoid

2. Draw a circuit containing multipliers, summers and digital inverters to obtain the BFSK signal $x_{BFSK}(t)$ from a NRZ_L baseband signal x(t) like the following



Make use of the fact that $x_{BFSK}(t) = x(t)A_c \cos(2\pi f_1 t) + \overline{x(t)}\,A_c \cos(2\pi f_2 t)$ where $\overline{x(t)}$ is the logical inverse of x(t)

3. For BFSK to work "best" we choose the frequencies f_1 and f_2 so the two sinusoids $s_1(t) = A_c \cos\left(2\pi f_1 t\right)$ and $s_2(t) = A_c \cos\left(2\pi f_2 t\right)$ are **orthogonal** - so they satisfy

$$\int_0^T s_1(t)s_2(t)dt = 0$$

where T is the time per bit. Show that $s_1(t)$ and $s_2(t)$ are orthogonal if $f_1 - f_2 = \frac{m}{T}$ and $f_1 + f_2 = \frac{n}{T}$ for integers m and n

4. The objective of this problem is to find and sketch the power spectral density of the BFSK signals from Problem (2) as follows

$$x_{BFSK}(t) = x(t)A_c \cos(2\pi f_1 t) + \overline{x(t)}\,A_c \cos(2\pi f_2 t)$$

   a. Make use of the fact that the autocorrelation $R_{BFSK}(\tau)$ of the BFSK signal can be shown to equal

$$R_{BFSK}(\tau) = E[s_{BFSK}(t)s_{BFSK}(t+\tau)] = \frac{A_c^2}{2} R_X(\tau)\cos(2\pi f_1\tau) + \frac{A_c^2}{2} R_X(\tau)\cos(2\pi f_2\tau)$$

plus our previous result that

$$F\left[\frac{A_c^2}{2} R_X(\tau)\cos(2\pi f_c\tau)\right] = \frac{A_c^2}{4}[S_X(f-f_c) + S_X(f+f_c)]$$

to find the power spectral density $S_{BFSK}(f)$. The answer contains 8 terms. Hint - make use of the result for $S_X(f)$ in Investigation 10

b. Sketch $S_{BFSK}(f)$ when $f_1 - f_2 = \frac{1}{T}$

5. What is the difference between a coherent and a noncoherent detector

6. Draw the block diagram of a noncoherent detector of BFSK signals using bandpass filters

7. Draw a block diagram of a coherent correlator detector for BFSK signals

8. Show that the average energy E_b of a BFSK signal is

$$E_b = \frac{A_c^2 T}{2}$$

9. Show that the probability of error P_e for a correlation BFSK receiver is given by

$$P_e = Q\left(\sqrt{\frac{A_c^2 T}{2N_o}}\right) = Q\left(\sqrt{\frac{E_b}{N_o}}\right)$$

where $Q(a) = \frac{1}{\sqrt{2\pi}}\int_a^\infty e^{-z^2/2}\,dz$ assuming the signals are orthogonal