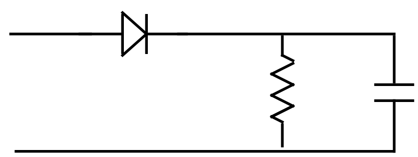
ECE 409 - BANDPASS TRANSMISSION - INVESTIGATION 11
INTRODUCTION TO AMPLITUDE SHIFT KEYING - PART II

SUMMER 2004 **A.P. FELZER**

To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

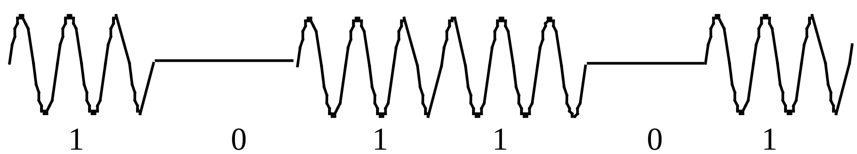
In the last Investigation we found the power spectral densities of BASK signals in order to find their bandwidths. The objectives of this Investigation are to introduce a decoder for BASK signals and to calculate the probability of bit error.

1. We begin with the finding of noncoherent detectors of BASK signals like those used to demodulate AM
   a. Explain what envelope detectors do. Draw a picture to illustrate
   b. Explain how envelope detectors introduced in ECE 405 as follows



   do what they do

   c. Sketch the response of an envelope detector to the following input

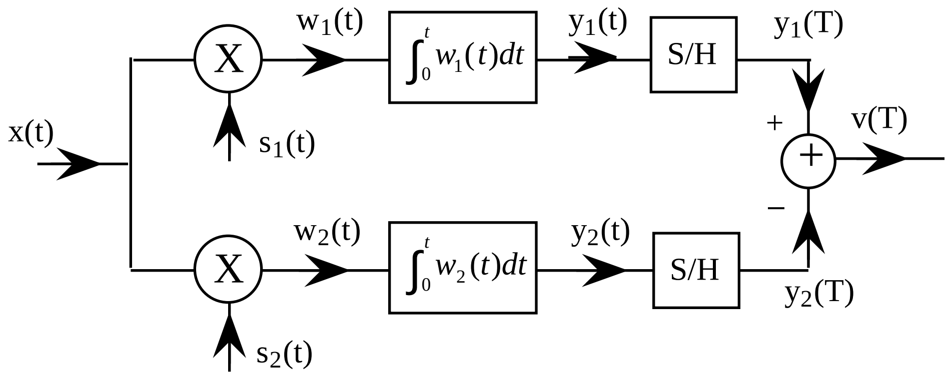


   d. Describe in words how an envelope detector can be used to differentiate between the 1's and 0's

2. From our Investigations on baseband digital signals we know that noise - in particular additive white Gaussian noise (AWGN) present in communication channels - can corrupt the signals s(t) representing the 1's and 0's. We also know that matched filters at the receiver with impulse responses $h(t)$ as follows

$$h(t) = s(T - t)$$

can minimize the expected value of the square of the error caused by the noise at the times T when s(t) is sampled. We also know that such matched filters are equivalent to correlation receivers as follows



The objective of this problem is to review how to find v(T) when $s_1(t)$ is the signal transmitted and how to find v(T) when $s_2(t)$ is transmitted

a. Show that when $x(t) = s_1(t) + n(t)$ then

$$v(T) = v_1(T) = s_{01}(T) + n_o(T)$$

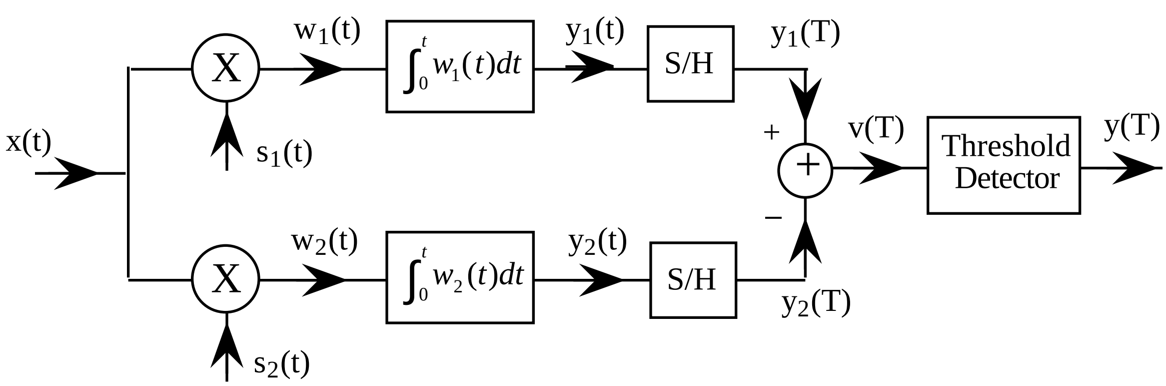
where $s_{01}(T) = \int_0^T s_1(t)[s_1(t) - s_2(t)]dt$ and $n_0(T) = \int_0^T n(t)[s_1(t) - s_2(t)]dt$

b. Show that when $x(t) = s_2(t) + n(t)$ then

$$v(T) = v_2(T) = s_{02}(T) + n_o(T)$$

where $s_{02}(T) = \int_0^T s_2(t)[s_1(t) - s_2(t)]dt$ and $n_0(T) = \int_0^T n(t)[s_1(t) - s_2(t)]dt$

3. For us to be able to differentiate between $s_1(t)$ and $s_2(t)$ in the following circuit from Problem (2) we would like the difference between $v(T)$ when $x(t) = s_1(t)$ and $v(T)$ when $x(t) = s_2(t)$ to be as large as possible



can decide which signal was sent. This can be done for BASK signals like ours with $s_1(t)$ equal to a sinusoid of frequency f_c by choosing f_c to be an integer multiple of $1/T$ as follows

$$f_c = \frac{m}{T}$$

where T is the length of time for each 1 and for each 0

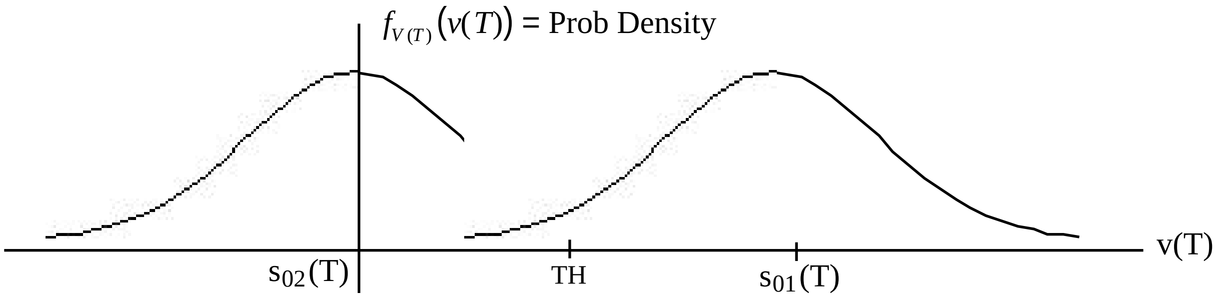
a. Find $v(T)$ when $x(t) = s_1(t) = A_c \cos(2\pi f_c t)$
b. Find $v(T)$ when $x(t) = s_2(t) = 0$

c. Describe how a detector can make use of the results of parts (a) and (b) to determine which signal was transmitted

4. From Problem (3) we know that if there's no noise in the channel then it's easy to determine whether the transmitted signal was $s_1(t)$ or $s_2(t)$. But the noise of course is going to change the values of the signals. In particular, as a result of the fact that the noise $n(t)$ in the channel is Gaussian it can be shown that

$$v_1(T) = s_{01}(T) + n_o(T) \quad \text{and} \quad v_2(T) = s_{02}(T) + n_o(T)$$

will have Gaussian probability densities as follows



with variances as follows

$$\sigma_o^2 = \frac{N_o}{2}\left[E_1 - 2\gamma\sqrt{E_1E_2} + E_2\right]$$

where N_o is the constant power spectral density of the white noise $n(t)$ and γ is the correlation coefficient of $s_1(t)$ and $s_2(t)$ as follows

$$\gamma = \frac{1}{\sqrt{E_1E_2}}\int_0^T s_1(t)s_2(t)dt$$

a. Sketch $f_{V(T)}(v(T))$ if the probability of $s_1(t)$ is larger than that of $s_2(t)$
b. Make use of your graph to explain why a threshold detector with threshold TH is sometimes going to make mistakes - why sometimes it's going to say the received signal is a 1 when it's really a 0 and vice versa
c. Sketch $f_{V(T)}(v(T))$ if the noise in the channel were less
d. Sketch $f_{V(T)}(v(T))$ if the energy of $s_1(t)$ was increased
e. Make use of the fact that for normalized Gaussian distributions

$$Q(a) = \frac{1}{\sqrt{2\pi}}\int_a^\infty e^{-z^2/2}\,dz$$

to show that when the 1's and 0's are equally likely then

$$P_e = Q\left(\sqrt{\frac{E_1 - 2\gamma\sqrt{E_1E_2} + E_2}{2N_o}}\right)$$

5. The objective of this problem is to apply our results to BASK signals. Given a BASK signal with equally likely signals for 1 and 0 equal to $s_1(t)$ and $s_2(t)$ as follows

$$s_1(t) = A_c \cos(2\pi f_c t) \quad \text{and} \quad s_2(t) = 0$$

with f_c an integer multiple of 1/T and

$$v_{TH} = \frac{s_{01}(T) + s_{02}(T)}{2}$$

a. Find $s_{01}(T)$ and $s_{02}(T)$
b. Find E_1 and E_2 of $s_1(t)$ and $s_2(t)$
c. Find γ
d. Make use of Problem (4e) to find P_e as a function of A_c, T and N_o
e. Make use of your result in part (d) to show that

$$P_e = Q\left(\sqrt{\frac{E_b}{N_o}}\right)$$

where E_b is the average energy per bit

6. Would you like your digital system to have $P_e = Q(a)$ or $P_e = Q(2a)$. Why. Illustrate with a graph of the Gaussian distribution