ECE 409 - BANDPASS TRANSMISSION - INVESTIGATION 10
INTRODUCTION TO AMPLITUDE SHIFT KEYING - PART I

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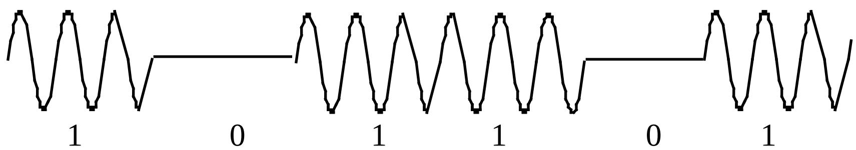
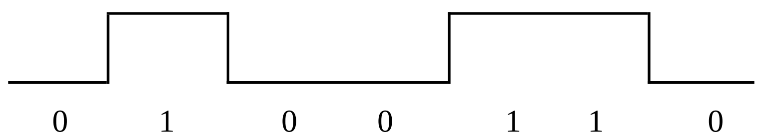
To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

In the previous group of Investigations we've seen what baseband digital signals look like in the time and frequency domains and we've seen how to calculate their bit error rates. In this and the next several Investigations we'll be calculating the spectrums and bit error rates of passband digital signals consisting of sinusoids with modulated amplitudes, frequencies and phases that can be transmitted through space as wireless transmission. We begin with amplitude shift keying.

1. **Amplitude Shift Keying (ASK)** in its most simple form gives us signals like the following for transmitting digital information



   We refer to such ASK signals as **Binary ASK (BASK)** or **On-Off Keying (OOK)**
   a. Describe the relationship between a binary signal and its BASK signal
   b. Draw the BASK signal y(t) corresponding to the following NRZ-L signal



   c. Describe how BASK signals can be obtained from NRZ-L signals

2. From Problem (1) we know that we can obtain the BASK signal y(t) for a given digital sequence by simply multiplying the corresponding NRZ-L signal x(t) by a sinusoid as follows

$$y(t) = x_{BASK}(t) = x(t)A_c\cos(2\pi f_c t)$$

   The objective of this problem is to find a general expression for the power spectral density of $x_{BASK}(t)$

   a. First show that the autocorrelation of $x_{BASK}(t)$ is given by

$$R_Y(t+\tau,t) = E\left[x_{BASK}(t+\tau)x_{BASK}(t)\right] = \frac{A_c^2}{2}R_X(\tau)\cos(2\pi f_c\tau) + \frac{A_c^2}{2}R_X(\tau)\cos(4\pi f_c t + 2\pi f_c\tau)$$

   b. Make use of your result in part (a) to show that $R_Y(t+\tau,t)$ is **cyclostationary** - that it's periodic in t with period T_o as follows

$$R_Y\left((t+T_o)+\tau,(t+T_o)\right) = R_Y\left(t+\tau,\,t\right)$$

   What is T_o

c. Make use of your result in part (b) to show that the time average of the autcorrelation $\langle R_Y(t+\tau,t)\rangle$ is equal to

$$\langle R_Y(t+\tau,t)\rangle = \frac{A_c^2}{2} R_X(\tau)\cos(2\pi f_c\tau)$$

Hint - make use of the fact that the average of a periodic signal is equal to the average over a single period

d. Now make use of the general form of the Wiener-Khinchine theorem for the power spectral density of a random process as follows

$$S_{BASK}(f) = F\left[\langle R_{BASK}(t,t+\tau)\rangle\right]$$

to show that

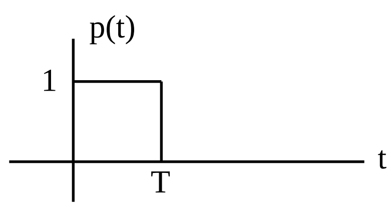
$$S_{BASK}(f) = \frac{A_c^2}{4}\left[S_X(f-f_c) + S_X(f+f_c)\right]$$

Hint - make use of Euler's Relation

3. From Problem (2) we see that to find the power spectral density of a BASK signal $x_{BASK}(t)$ we first need to find the autocorrelation $R_X(\tau) = E\left[x(t)x(t+\tau)\right]$ of the corresponding NRZ-L signals. Now as we saw in the previous Investigations on baseband digital signals the autocorrelation of a baseband NRZ-L signal as follows

$$x(t) = \sum_{n=-\infty}^{\infty} a_n p(t-nT)$$

with pulses $p(t)$ as follows



is given by

$$R_X(\tau) = E\left[x(t)x(t+\tau)\right] = \frac{1}{T}\sum_{m=-\infty}^{\infty} R_A(m)R_P(\tau - mT)$$

where $R_A(m) = E\left[a_n a_{n+m}\right]$ = autocorrelation of the random sequence a_n

$R_p(\tau) = \int_{-\infty}^{\infty} p(t)p(t+\tau)dt$ = autocorrelation of the deterministic pulse p(t)

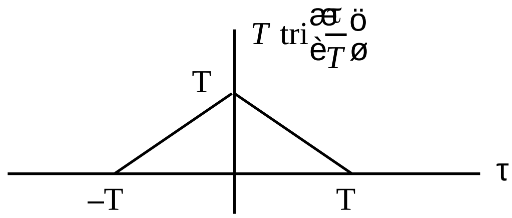
a. First show that if the a_n's are arbitary sequences of 1's and 0's with P(0)=P(1)=0.5 then

$$R_A(m) = \begin{cases} \frac{1}{2} & m = 0 \\ \frac{1}{4} & m \neq 0 \end{cases}$$

b. Now make use of your result in part (a) to show that

$$R_X(\tau) = \frac{1}{4T} R_p(\tau) + \frac{1}{4T} \sum_{m=-\infty}^{\infty} R_p(\tau - mT)$$

c. Make use of the fact that the autocorrelation of a pulse $R_p(\tau)$ is a triangular function as follows

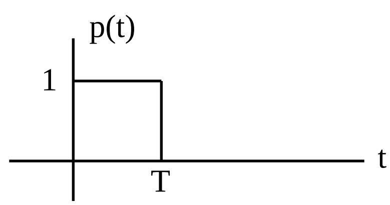


to sketch $R_X(\tau)$. Hint - First sketch $\frac{1}{4T} R_p(\tau)$, then sketch $\frac{1}{4T} \sum_{m=-\infty}^{\infty} R_p(\tau - mT)$ and then add the two together

4. Taking the Fourier Transform of $R_X(\tau)$ in Problem (3) we obtain the power spectral density for the NRZ-L signal x(t) as follows

$$S_X(f) = F\left[R_X(\tau)\right] = \frac{1}{4T}\left|P(f)\right|^2 + \frac{1}{4T^2} \sum_{n=-\infty}^{\infty} \left|P(nf_c)\right|^2 \delta(f - nf_c)$$

a. Make use of the fact that the Fourier Transform of a pulse of amplitude one as follows



is equal to

$$P(f) = T\,\mathrm{sinc}(Tf)$$

to show that $S_X(f)$ for an NRZ-L signal is given by

$$S_X(f) = \frac{T}{4}\mathrm{sinc}^2(Tf) + \frac{1}{4}\delta(f)$$

when the width T of the pulse is an integer multiple of $T_c = 1/f_c$

b. Make use of your result in part (a) to sketch $S_X(f)$

5. Now that we have an epxression for $S_X(f)$ from Problem (4)
   a. Make use of the result in Problem (2) to find $S_{BASK}(f)$
   b. Make use of your result in part (a) to sketch $S_{BASK}(f)$