ECE 409 - INTROD TO DIGITAL COMMUNICATIONS - INV 1
INTRODUCTION TO DIGITAL COMMUNICATIONS - PART I

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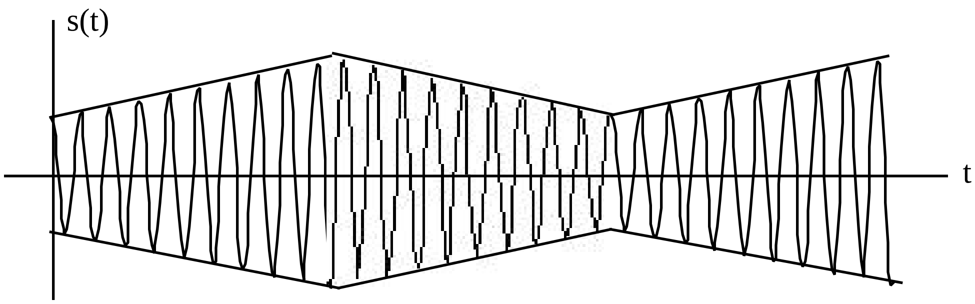
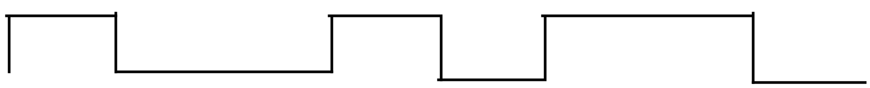
To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

In ECE 405 we learned about analog communication systems like AM and FM in which message signals m(t) modulate carrier signals $c(t) = A_c \cos(2\pi f_c t)$ by varying their amplitudes and frequencies like in the following AM signal



In digital communication systems we're still transmitting analog signals like the following pulse train



but now the pulses are for the 1's and 0's we obtain when we convert samples of the message signal to binary. We call this **pulse code modulation (PCM)**. Now PCM can involve a fair amount of processing and can take a fair amount of bandwidth. But it's all more than compensated for by the capabilities of digital integrated circuits.

Our main goal in this class is to introduce various ways to transmit digital information including baseband transmission, passband transmission and spread spectrum (CDMA). We will be particularly interested in the bandwidths of these signals and their bit error rates as well as how we can deal with the limitations of real communications channels. In this and the next Investigation we review **source coding** including sampling theory, conversion of sample values to binary and compression

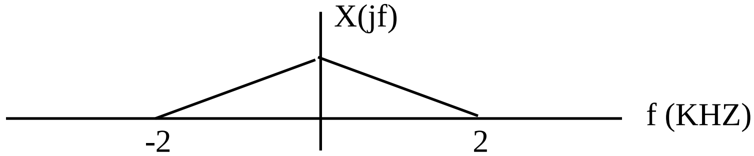
1. From ECE 405 we know that we can recreate a **bandlimited** analog signal x(t) from its samples if the sampling frequency f_s is sufficiently large as follows

$$f_s > 2 f_b$$

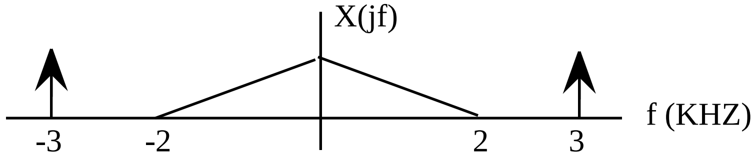
   where f_b is the highest frequency in x(t). Draw the spectrum (Fourier Transform) of a signal $x(t)$ that is bandlimited by $f_b = 2\ KHz$

2. How fast do we need to sample the following signals to be able to recreate them from their samples

a. $x(t) = 2\cos(2\pi 1000t)$
b. $x(t) = 2\cos(2\pi 1000t) + 3\cos(2\pi 2000t)$
c. $x(t) = 2\cos(2\pi 1000t)\cos(2\pi 2000t)$
d.



e.



3. Suppose we sample the following signal $x(t)$ at $f_s = 3500$ samples/sec

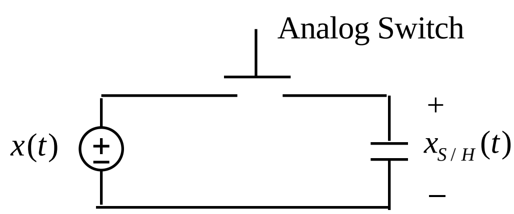
$$x(t) = 2\cos(2\pi 1000t)$$

   a. What is the sample time T_s - the time between samples
   b. Find the first 5 samples starting at $t = 0$

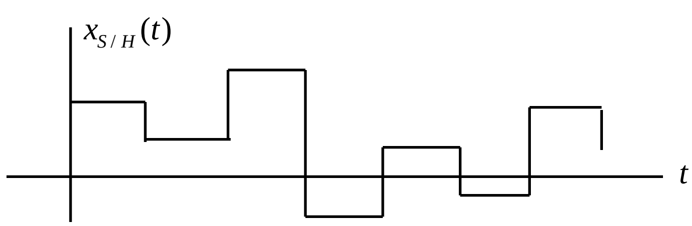
4. Graph sample time T_s as a function of sampling frequency f_s. Describe what's going on

5. What happens when we don't sample fast enough

6. Circuits like the following



with outputs $x_{S/H}(t)$ as follows



are referred to as **sample-and-hold (S/H) circuits**.
   a. Explain how our simple sample-and-hold circuit works - how it's able to sample and then hold $x(t)$
   b. Sketch a timing diagram for $0 \le t \le 2$ msec that includes the clock controlling the analog switch and $x_{S/H}(t)$ when $x(t) = 2\cos(2\pi 1000t)$ sampled at $f_s = 5000$ samples/sec

7. Once a message signal $m(t)$ has been sampled we convert it to binary with an **analog-to-digital converter (ADC)** as follows

where the ADC is basically a circuit of voltage dividers and comparators. The simplest binary code produced by such circuits is signed binary like the following

$$A = 01100101$$

where the MSB tells us the sign of A as follows

MSB	*Sign*
0	+
1	–

Express the following signed binary numbers in decimal

a. $A = 01101101_{SB}$
b. $B = 11101101_{SB}$

8. Convert each of the following decimal numbers to 8-bit signed binary

a. $A = 734$
b. $B = -734$

9. A general method for converting samples m_s in the range

$$-m_{\max} \le m_s \le m_{\max}$$

to n-bit signed binary is as follows

(1) Calculate the resolution $\Delta = \dfrac{2m_{\max}}{2^n} = \dfrac{m_{\max}}{2^{n-1}}$

(2) Calculate $m_\Delta = \dfrac{m_s}{\Delta}$ rounded off to the nearest integer

(3) Convert m_Δ to signed binary

Assuming $m_{\max} = 10$

a. Convert $m_s = 7.34$ to 8-bit signed binary
b. Convert $-m_s = 7.34$ to 8-bit signed binary
c. Find m_s in decimal if in 8-bit signed binary $m_s = 01011101_{SB}$

10. Whenever we convert a sample value to binary there will usually be roundoff or truncation error as follows

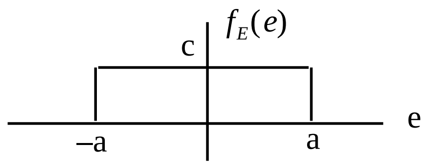
$$\text{Conversion error} = \text{Sample value} - \text{Binary value}$$

a. Find the roundoff error when $m_s = 7.37$ is converted to 8-bit signed binary
b. Suppose we convert samples to n-bit signed binary with a resolution of Δ. Explain why the roundoff error e is in the range

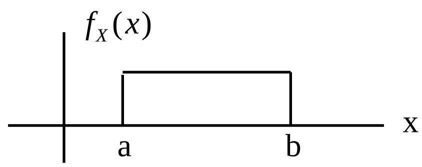
$$-\frac{\Delta}{2} \le e \le \frac{\Delta}{2}$$

c. What happens to the roundoff error when we add an extra bit

11. Assuming that roundoff error as discussed in the last problem has a uniform probability density as follows



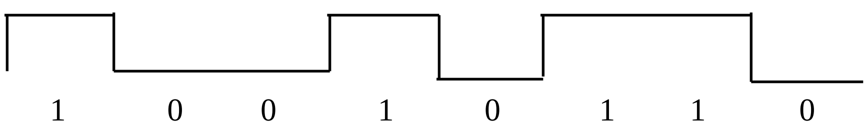
a. Find a and c if samples $-5 \le m_s \le 5$ are converted to 8-bit signed binary numbers
b. Find the average of the square of the roundoff in part (a). Hint - make use of the result from ECE 315 that if a random variable X has a uniform probability density as follows



then its variance is given by $Var[x] = \frac{(a-b)^2}{12}$

12. Under what circumstances are samples m_s converted to 2's complement

13. Once we've converted the sample values to 1's and 0's - converted the signal to a pulse code modulated (PCM) signal - we can then transmit them as **baseband signals** like the following pulse train



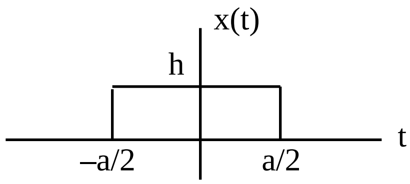
which is referred to as a **Not-Return-To-Zero (NRZ)** signal with pulses for 1's and no pulses for 0's. Or we can use the 1's and 0's to generate **passband signals** obtained by modulating the amplitude, frequency and/or phase of a sinusoid

a. Sketch the NRZ baseband signal for $A = 1101001$
b. Sketch a passband signal for $A = 1101001$ consisting of a sinusoid with a high frequency for 1 and a low frequency for 0

14. Circuit Analysis Review - What do we mean by the frequency response of a circuit

15. Fourier Transform Review - Find and sketch the Fourier Transforms of the following signals

a.



b. $x(t) = A\cos(2\pi bt)$

16. Convolutiuon Review - Sketch the convolution of $x(t)$ and $y(t)$ as follows