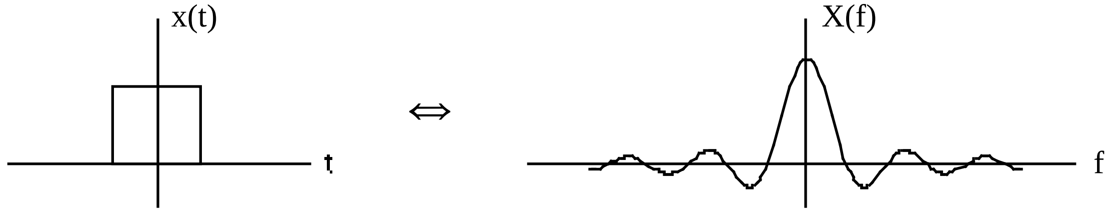
# ECE 405 - ANALOG COMMUNICATIONS - INVESTIGATION 6
# INTRODUCTION TO AMPLITUDE MODULATION - PART I

**FALL 2005** **A.P. FELZER**

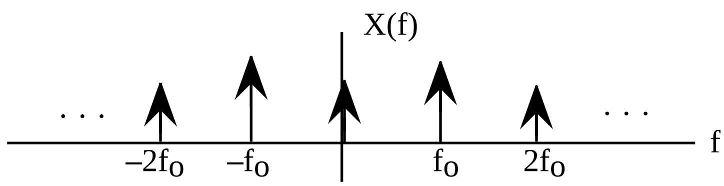
To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

From the last two Investigations on Fourier Transforms we have that

(1) The Fourier Transform of a pulse is a sinc as follows



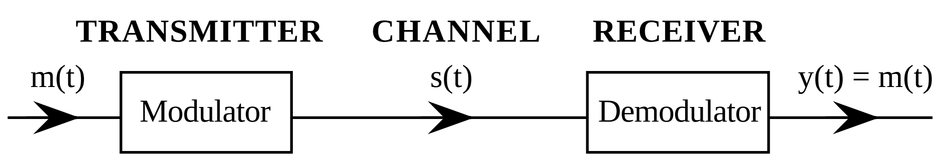

(2) The Fourier Transform of a periodic signal $x(t)$ like a pulse train is a sequence of impulses as follows




(3) The Fourier Transform of a product is the convolution of the Fourier Transforms as follows

$$F[x(t)y(t)] = X(f) \quad Y(f)$$

The objective of this and the next several Investigations is to make use of these results to introduce analog communication systems of the following form

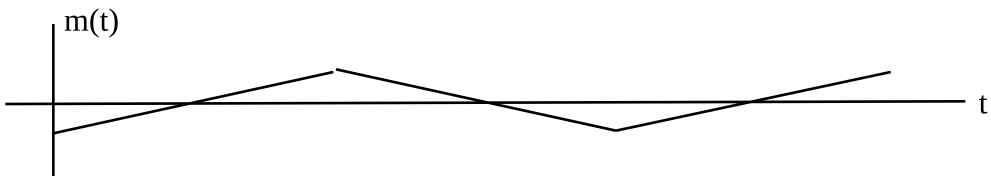



consisting of a transmitter, a channel like space or fiber optic and a receiver.

The **modulator** is a circuit in the transmitter that "shifts" the analog input m(t) consisting of signals like voice and music in the 0-20KHz range to *higher frequencies* for transmission over the communication channel. We do this for several reasons - one of the most important being that signals transmitted through space must be at least in the megahertz range for the size of the antennas to be small enough to be practical.

The **demodulator** is a circuit in the receiver that recovers m(t) from s(t). Note that we refer to the input m(t) as a **baseband** signal and the modulated signal s(t) as a **passband** signal. Note also that the communication channel can be copper wire, fiber optic, free space or any number of other possibilities.

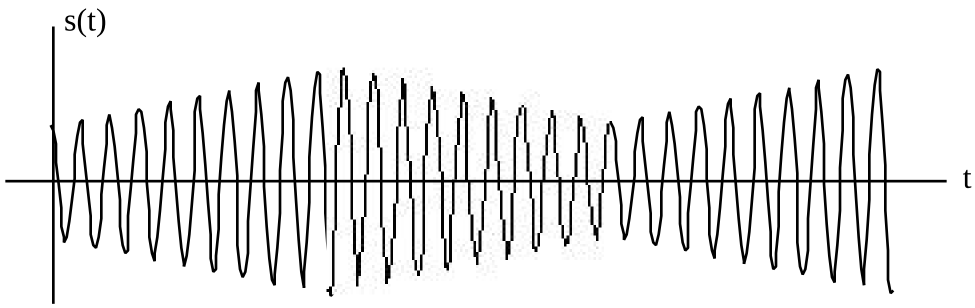
As we will see there are a number of different ways to modulate signals. The objective of this investigation is to introduce what we refer to as **amplitude modulation (AM)** where input signals m(t) like the following



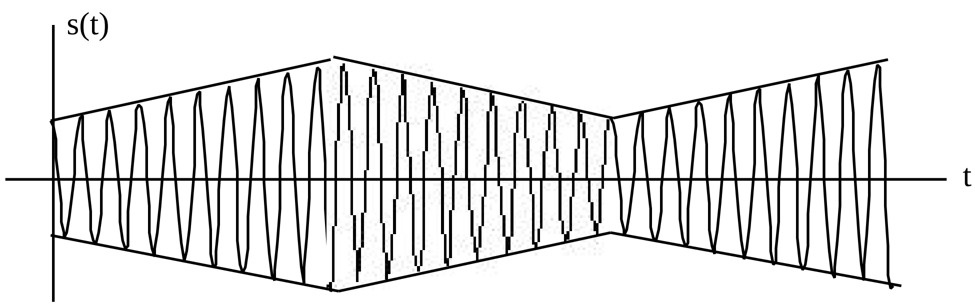

are multiplied times *much higher* frequency sinusoids called **carrier signals** as follows

$$c(t) = A_c \cos(2\pi f_c t)$$

to generate sinusoids that vary in amplitude in accordance with $m(t)$ as follows




Note that the curves going through the peaks of the sinusoids as follows

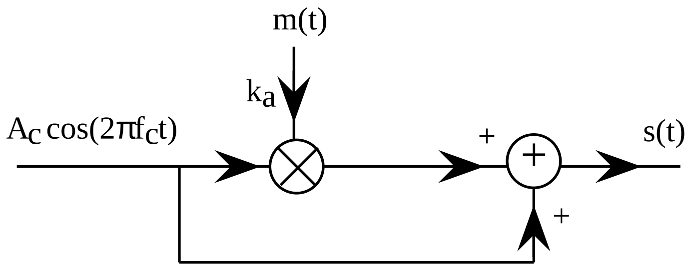



are referred to as the **envelope** of the amplitude modulated signal.

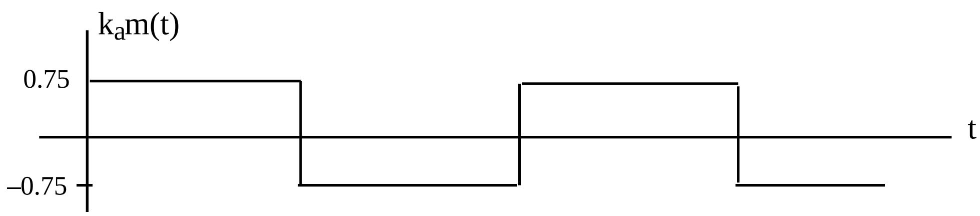
1. The objective of this first problem is to sketch some AM signals $s(t)$ of the form

$$s(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)$$

   a. First multiply everything out and then describe in words what $s(t)$ is the sum of
   b. Then verify that $s(t)$ can be generated by a system with the following block diagram




c. Given $k_a m(t)$ equal to the following pulse train




(i) Sketch $1 + k_a m(t)$
(ii) Sketch $s(t)$

d. Given $k_a m(t) = 0.5\cos(2\pi f_m t)$
(i) Sketch $1 + k_a m(t)$
(ii) Sketch $s(t)$

2. The objective of this problem is to find the condition on the **coefficient of modulation $k_a$** so that the envelope of the following AM signal

$$s(t) = A_c\left[1 + k_a \cos(2\pi f_m t)\right]\cos(2\pi f_c t)$$

will be proportional to $m(t)$ in the special case when $m(t) = \cos(2\pi f_m t)$. Now assuming $f_c >> f_m$

a. Sketch $s(t)$ and its envelope when $k_a = 0.5$
b. Sketch $s(t)$ and its envelope when $k_a = 1$
c. Sketch $s(t)$ and its envelope when $k_a = 1.5$
d. What can you conclude from your results in parts (a) - (c)

3. Generalizing on the result of Problem (2) we have that if s(t) is an AM signal as follows

$$s(t) = A_c\left[1 + k_a m(t)\right]\cos(2\pi f_c t)$$

with m(t) normalized to one as follows

$$|m(t)|_{\max} = 1$$

then the envelope of s(t) will be proportional to m(t) as long as $k_a$ satisfies

$$0 < k_a < 1$$

This result is very important because the *big advantage* of AM is how simply we can obtain $m(t)$ from $s(t)$ when the envelope of $s(t)$ is proportional to $m(t)$

a. Will the envelope of $s(t) = \left[1000 + 700\cos\left(2\pi f_m t\right)\right]\cos\left(2\pi f_c t\right)$ be proportional to $m(t) = \cos\left(2\pi f_m t\right)$. How can you tell
b. Find an equation for the maximum and minimum of the envelope of $s(t) = A_c\left[1 + k_a m(t)\right]\cos\left(2\pi f_c t\right)$ when $0 < k_a < 1$ assuming that $|m(t)|_{max} = 1$
c. Use your result in (b) to sketch the envelope of $s(t) = A_c\left[1 + 0.5\cos\left(2\pi f_m t\right)\right]\cos\left(2\pi f_c t\right)$
d. Verify that your graph of the envelope in part (c) is equal to the envelope of s(t) in Problem (2a)
e. Why is $k_a = 1$ referred to as 100% modulation

4. The objective of this and the next two problems is to find the spectrums of amplitude modulated signals when the message signals are sinusoids. Given the following AM signal

$$s(t) = 5\left[1 + 0.6\cos(2\pi 1000t)\right]\cos\left(2\pi 10^6 t\right)$$

with baseband signal m(t) = cos (2π1000t)

a. Find and sketch the spectral plot of m(t)
b. Find and sketch the spectral plot of the amplitude modulated signal s(t)
c. What is the bandwidth of s(t)
d. Describe how the spectrum of s(t) is related to the spectrum of m(t)
e. Find the average normalized power of m(t)
f. Find the average normalized power of s(t)
g. Compare the average normalized powers of s(t) and m(t)

5. The objective of this problem is to find spectrum of the following AM signal with baseband signal equal to the sum $m(t) = 0.5\cos(2\pi 1000t) + 0.5\cos(2\pi 2000t)$

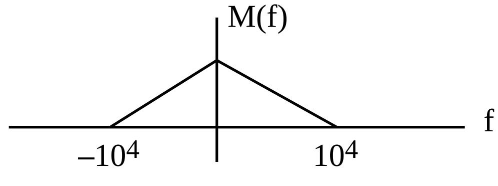
$$s(t) = 5\left[1 + 0.8\left(0.5\cos(2\pi 1000t) + 0.5\cos(2\pi 2000t)\right)\right]\cos\left(2\pi 10^6 t\right)$$

a. Find and sketch the spectral plot of m(t)
b. Find and sketch the spectral plot of the amplitude modulated signal s(t)
c. What is the bandwidth of s(t)
d. Describe how the spectrum of s(t) is related to the spectrum of m(t)

6. The objective of this problem is to generalize on the results of Problems (4) and (5) to where m(t) is a nonperiodic signal. In particular given the following AM signal

$$s(t) = 5\left[1 + k_a m(t)\right]\cos\left(2\pi 10^6 t\right) = 5\cos\left(2\pi 10^6 t\right) + 5k_a m(t)\cos\left(2\pi 10^6 t\right)$$

where m(t) has the spectrum M(f) as follows




a. Make use of the fact that the Fourier Transform of a product is the convolution of the Fourier Transforms as follows

$$F\left[m(t)\cos\left(2\pi f_o t\right)\right] = M(f) \quad \left(0.5\delta\left(f - f_o\right) + 0.5\delta\left(f + f_o\right)\right)$$

to sketch the spectrum of s(t)

b. What is the bandwidth of s(t)
c. Describe how the spectrum of s(t) is related to the spectrum of m(t).
d. Note that the spectrums above and below the center frequency are referred to as **sidebands**. **Memorize** this term. Then describe the relationship between the sidebands of an AM signal

7. Math Review -
    a. Express $z = 5e^{j2.2}$ in rectangular form
    b. Find $x(t) = \mathrm{Re}\left[5e^{j(2\pi 2000t+1.2)}\right]$
    c. What is the period of $z(t) = 5e^{j(2\pi 2000t+1.2)}$