ECE 405 - BASEBAND TRANSMISSION - INVESTIGATION 22
NOISE IN CENTER POINT DETECTION

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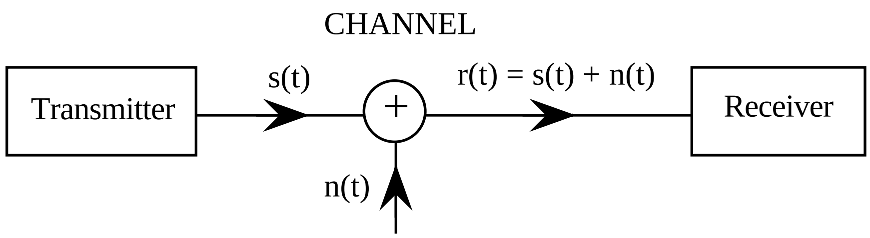
To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

In the previous Investigations we showed how raised cosine pulses can reduce intersymbol interference and we showed how equalizers can reduce the distortion caused by the nonideal frequency responses of real communication channels. The objective of this and the next two Investigations is to develop a method for minimizing the affects of the channel noise $n(t)$ that gets added to and as a result distorts the transmitted signal $s(t)$ as follows



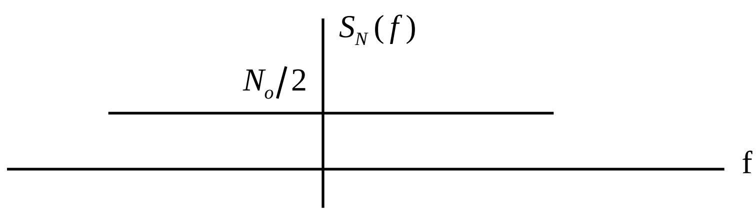
The objective of this Investigation in particular is to characterize the channel noise and see how it affects the ability of the receiver to distinguish 1's from 0's when *center point detection* is used

1. Suppose we're transmitting a baseband signal of rectangular pulses of duration T with amplitudes 1 and –1 as follows

bit	*amplitude*
0	−1
1	1

   Assuming the 1's and 0's are equally likely sketch a typical signal if there's
   a. No noise
   b. A small amount of zero mean noise
   c. A larger amount of zero mean noise

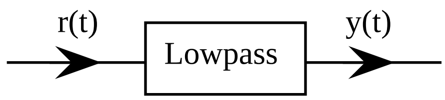
2. Suppose we're transmitting raised cosines with $r = 1$ and bandwidths $f_b = 1/T$ and that the noise in the channel, as usual, can be approximated as zero mean white Gaussian noise. This means that the power spectral density of the noise is constant as follows



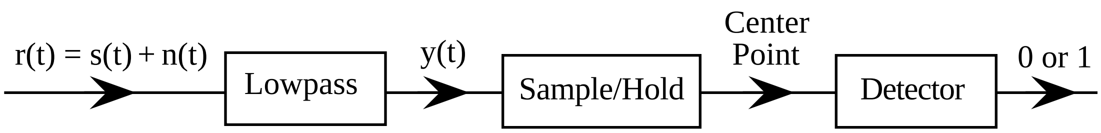
   and the probability distribution of the amplitudes is Gaussian with zero mean
   a. Sketch the frequency response of an ideal lowpass filter that just passes the pulses

b. What's the advantage of putting a lowpass filter like the one in part (a) at the input of the receiver. Hint - sketch the power spectral density of the noise that makes it through the lowpass filter
c. Find the average power of the noise that makes it through the lowpass filter in terms of N_o and T

3. Once the high frequency noise has been removed from the received signal $r(t)$ as follows



the receiver then has to decide - every T seconds - whether $y(t)$ is the signal of a 1 or of a 0. Let us suppose our receiver is using a *center point detector* as follows



with a detector that makes its decision as follows

$y(nT)$	*Decision*
< 0	0
> 0	1

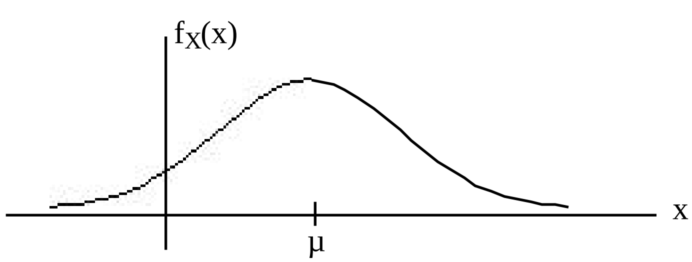
a. Explain in words why we call 0 the *threshold* of this detector. **Memorize** your definition
b. Draw a circuit for implementing the detector

4. Explain how the noise can cause the detector to make a mistake. Illustrate with a drawing of a received baseband signal with noise that has caused at least one error

5. To find the probability that the detector makes a mistake we need to review the Gaussian distribution as follows

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

which can be graphed as follows



a. Sketch $f_X(x)$ for $\mu > 0$
b. Describe in words what the graph of $f_X(x)$ looks like
c. What is the area under the curve $f_X(x)$

d. What does μ tell us about the graph
e. What does the variance σ^2 tell us about the graph
f. Sketch on the same graph two Gaussian density functions with the same variances but different means
g. Sketch on the same graph two Gaussian density function with mean zero but different variances. Label the curve with the larger variance
h. Describe the differences between your graphs in part (f)

6. Given the following probability P

$$P = \int_a^b f_X(x)dx = \int_a^b \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

a. Describe in words what P is the probability of
b. Show the region being integrated under the Gaussian density curve

7. Make use of the fact that

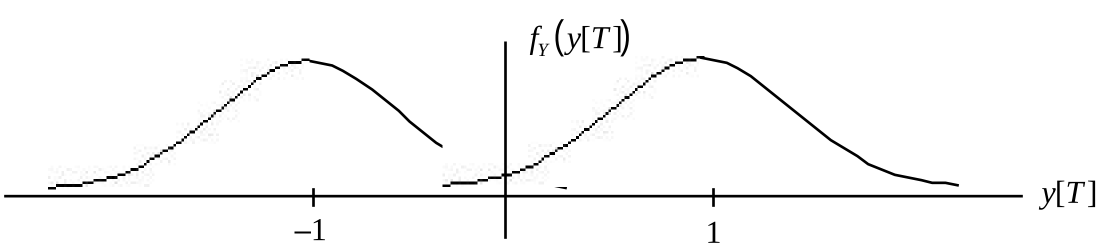
$$P(a \le x \le b) = \int_a^b \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = \int_{\frac{a-\mu}{\sigma}}^{\frac{b-\mu}{\sigma}} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz$$

where $f_Z(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$ is the normalized Gaussian distribution to find $P(2 \le x \le 5)$ for a Gaussian distribution with $\mu = 3$ and $\sigma = 2$.

8. Find $P[x \ge 7]$ for a random variable X with a Gaussian distribution with $\mu = 5$ and $\sigma = 2.5$

9. Assuming the 1's and 0's being transmitted in Problem (1) are equally likely and the noise is zero mean Gaussian, sketch the probability densities of $y[T]$ if there is
   a. No noise
   b. A little noise
   c. A fair amount of noise

10. Given a graph of the probability density of $y[T]$ like the ones in Problem (9) as follows



a. What is the area under each curve
b. Use these curves to explain in words when the detector is making an error
c. Shade in the region corresponding to when errors are being made

11. Find the probability of an error in Problem (10) if the variance of the noise is $\sigma^2 = 0.2$

12. The probability of an error in a communication system is typically expressed in terms of Q(a) as follows

$$Q(a) = \int_b^\infty \frac{1}{\sigma_n\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma_n^2}}\, dx = \frac{1}{\sqrt{2\pi}}\int_a^\infty e^{-z^2/2}\, dz \quad \text{with} \quad a = \frac{b-\mu}{\sigma_n}$$

a. Find a when $b = 5$, $\mu = 2.7$ and $\sigma = 1.2$
b. Find Q(a) in part (a)

13. Now suppose we have a baseband signal of equally likely 0's and 1's made up of pulses of duration T and amplitudes A and $-A$ as follows

bit	*amplitude*
0	$-A$
1	A

a. Sketch the probability density of the amplitudes
b. Find the probability of error in terms of Q as a function of the pulse amplitude A, N_o and T assuming the bandwidth is $1/T$. Note that the variance of zero mean noise is equal to its average power