ECE 405 - BASEBAND TRANSMISSION - INVESTIGATION 20
INTRODUCTION TO INTERSYMBOL INTERFERENCE

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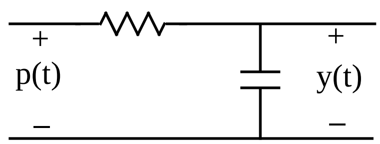
To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

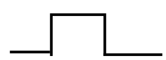
In the last Investigation we introduced various pulse train baseband signals. The objective of this Investigation is to first show how the limited bandwidths of communication channels like the following

distort these pulse trains and then show how the affect of this distortion can be minimized.

1. We begin by analyzing the response of a channel that can be modeled by a simple first order lowpass circuit as follows

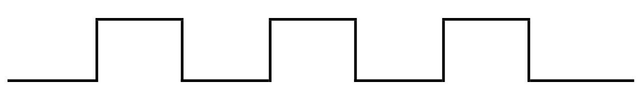


to a single pulse $p(t)$ as follows



a. Sketch the pulse response $y(t)$ of the circuit if the width of the pulse is 2τ
b. Describe the distortion - how $y(t)$ looks different from $p(t)$
c. Make use of time domain arguments - arguments that take into account the charging and discharging of the capacitor - to explain the distortion

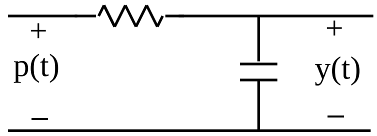
2. From Problem (1) we know that each pulse has a transient response. So if we now transmit three pulses like the following



over a communication channel then the still present transient response of the first pulse will interfere with the channel's response to the second and third pulses and so on. We call the resulting distortion **intersymbol interference (ISI)**. Assuming a pulse width of 3τ
a. Sketch the received signal if the time between pulses is equal to 5τ
b. Sketch the received signal if the time between pulses is equal to τ
c. Describe the differences between the responses
d. In which case is intersymbol interference the most

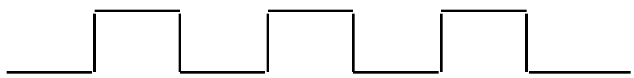
3. In Problems (1) and (2) we made use of time domain analysis to explain intersymbol interference in baseband signals. The objective of this problem is to make use of frequency domain analysis to come up with another way of looking at the problem. Suppose in particular that we have a nice simple channel like the one we've been working with that can be modeled by

a first order RC circuit as follows



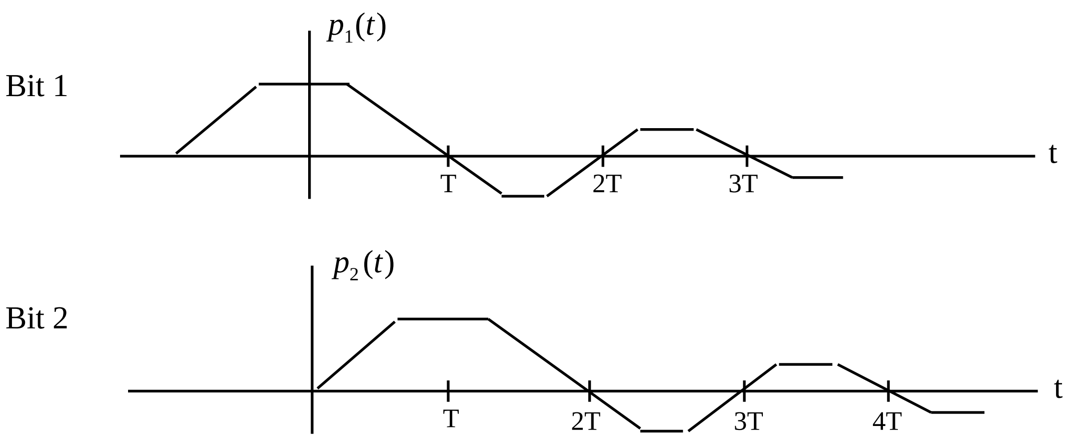
a. Sketch and describe the frequency response of the RC circuit
b. How large are the bandwidths of pulse trains $p(t)$
c. Make use of your results in parts (a) and (b) to explain why $y(t)$ has intersymbol interference

4. From Problem (3) we know that intersymbol interference in a channel is caused by trying to put pulse trains with infinite bandwidths like the following



through channels with finite bandwidths. So it looks like the problems of intersymbol interference (ISI) are insurmountable - unless of course we keep the pulse rate small enough to minimize the problem. Or equivalently use a channel with a really large bandwidth like fiber optic.

But in fact ISI can be greatly reduced with systems using pulses like the following



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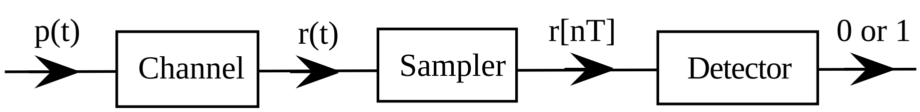
together with receivers built as follows

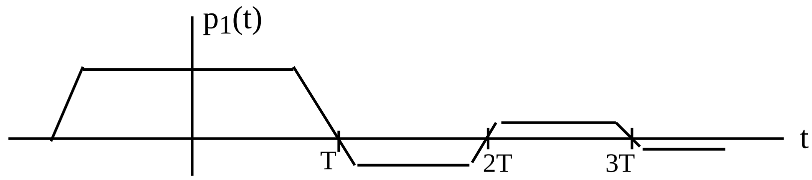


where $p(t) = p_1(t) + p_2(t) + \cdots$. The basic operation of such systems is as follows

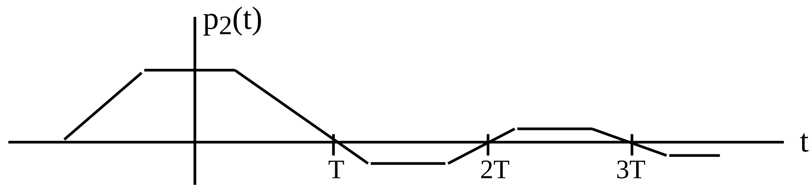
(1) The receiver samples the value of the incoming signal every T seconds
(2) The detector makes use of the sample value $r[nT]$ to decide whether the n'th bit is a 0 or a 1. Note that we refer to this as **center point detection** because the detector is

basing its decision on the value of the received signal at the center of the pulse. **Memorize** this term.

a. Explain how the above center point detection system minimizes the affects of intersymbol interference
b. Which of the following two pulses would do a better job of minimizing intersymbol interference in the real world



or



Hint - there will always be some jitter - there will always be some variations in the times the pulses are detected as a result of variations in the clocks.

5. From Problem (4) we know that we can reduce intersymbol interference if we use pulses that are zero every T seconds as follows

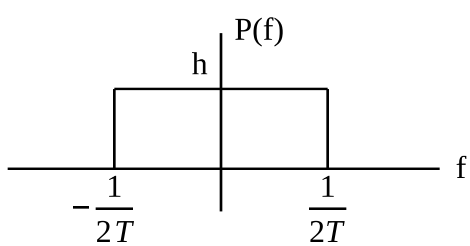
$$p(t) = \begin{cases} 1 & t = 0 \\ 0 & t = nT \end{cases}$$

when subsequent pulses are being detected. But the big question remains - can we find pulses $p(t)$ that not only satisfy this criteria but are also bandlimited and so won't be distorted by the finite bandwidths of our communication channels. The answer - as you've undoubtedly guessed - is yes. We call such bandlimited pulses **Nyquist pulses**. **Memorize** this term.

The "simplest" Nyquist pulses are sinc pulses as follows

$$p(t) = \frac{h}{T}\,\mathrm{sinc}\left(\frac{t}{T}\right)$$

with spectrums of bandwidth $BW = \frac{1}{2T} = \frac{1}{2} f_b$ as follows



a. Sketch $p(t)$ obtained by taking the inverse Fourier Transform of $P(f)$
b. Verify that $p(t)$ is a Nyquist pulse

6. Generalizing on the results of Problem (5) it can be shown that if we wish to transmit data at the rate of f_b bits/sec with **Nyquist pulses** - pulses of finite bandwidth that do not have intersymbol interference at the sampling times then the

   (1) Nyquist pulses must have bandwidths of at least $f_b/2$

   (2) Pulses $p(t)$ with bandwidths BW in the range

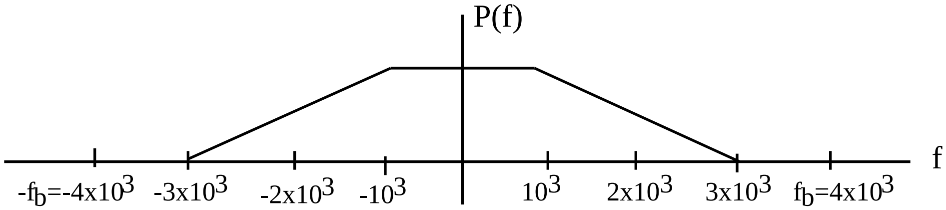
$$\frac{f_b}{2} \le BW \le f_b \qquad T = \frac{1}{f_b}$$

   that satisfy the condition

$$P(f + f_b) + P(f) + P(f - f_b) = K \qquad -f_b \le f \le f_b$$

   can be shown to be Nyquist pulses. Note that all Nyquist pulses that meet these conditions are said to have Nyquist frequency spectrums

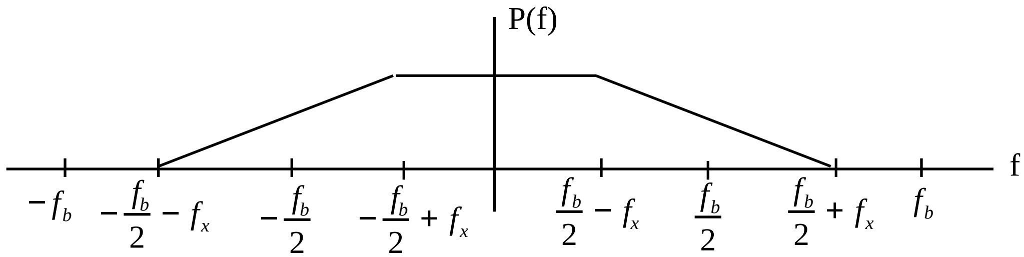
   a. Verify that the following spectrum satisfies the conditions of a Nyquist spectrum



   by graphing $P(f + f_b) + P(f) + P(f - f_b)$ in the range $-f_b \le f \le f_b$

   b. Come up with your own example of a Nyquist spectrum

7. If we now draw Nyquist spectrums P(f) as follows



   for pulses p(t) being transmitted at the rate

$$f_b = \frac{1}{T_b} \quad \text{bits/sec}$$

   then the bandwidth B_T needed to transmit these pulses without intersymbol interference is equal to

$$B_T = \frac{f_b}{2} + f_x$$

   If we now define the **roll-off factor r** as follows

$$r = \frac{f_x}{f_b/2} \qquad \text{then} \qquad B_T = (1+r)\frac{f_b}{2}$$

a. Does a small r mean a slow or fast rolloff of the frequency response
b. Find the bandwidth B_T required for transmitting pulses at a rate of 1.2×10^4 bits/sec with a roll-off factor of $r = 0.7$
c. What's the highest bit rate for pulses being transmitted over a channel with bandwidth $B_T = 20\text{KHz}$ for a roll-off factor of $r = 0.6$

8. A class of Nyquist pulses called **raised-cosine** pulses as follows

$$P(f) = \begin{cases} 1 & \text{when } |f| < \dfrac{f_b}{2} - f_x \\ \dfrac{1}{2}\left(1 - \sin\left[\dfrac{\pi\left(f - \dfrac{f_b}{2}\right)}{2 f_x}\right]\right) & \text{when } \left|f - \dfrac{f_b}{2}\right| \le f_x \\ 0 & \text{when } |f| > \dfrac{f_b}{2} + f_x \end{cases}$$

are particularly popular because of their relatively smooth frequency responses

a. Use Mathcad, Matlab or an equivalent to obtain plots of P(f) for roll-offs of $r = 0$, 0.5 and 1. Hint - choose a value for f_b and then calculate f_x from r
b. Describe how the plots of $P(f)$ change as the roll-off r increases
c. Verify that when $r = 1$ and $f_x = f_b/2$ then $P(f)$ becomes

$$P(f) = \frac{1}{2}\left(1 + \cos\left(\frac{\pi f}{f_b}\right)\right) \quad \text{for } |f| \le f_b$$

d. Verify that raised cosine pulses $p(t)$ for $r = 1$ as follows

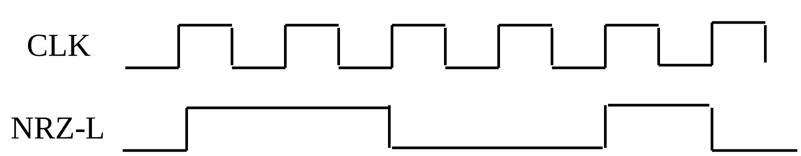
$$p(t) = f_b \frac{\cos(\pi f_b t)}{1 - 4 f_b^2 t^2} \operatorname{sinc}(f_b t)$$

obtained by taking the inverse Fourier Transform $P^{-1}(f)$ are zero at times nT_b for $n > 0$

9. At what rates can bits be transmitted through a channel with bandwidth $BW = 10^7$ Hz if raised cosines are used with rolloffs of
a. $r = 0$
b. $r = 0.5$
c. $r = 1$

10. Describe how a lookup table can be used to generate raised-cosine pulses

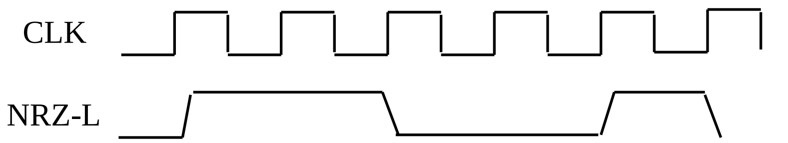
11. The objective of this problem is to introduce **eye patterns** for displaying distortion caused by intersymbol interference. If we display an ideal NRZ-L signal as follows



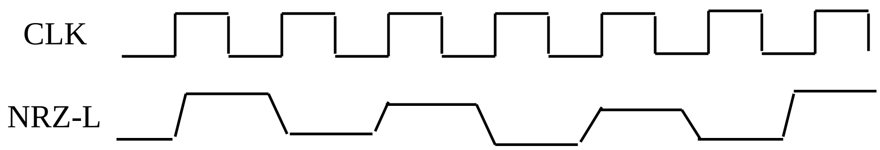
on a scope externally triggered by the rising edge of the clock pulses then we'll see a nice "clean" rectangular pattern as follows

We call such a signal an eye pattern because as the NRZ-L becomes distorted the signal displayed on the scope will look more and more like an eye

a. Sketch the eye pattern of the following NRZ-L signal with finite rise and fall times



b. Sketch the eye pattern of the following NRZ-L signal as follows with not only finite rise and fall times but also distorted amplitudes



c. Sketch the eye pattern of a received NRZ-L signal with a lot of distortion
d. Make use of your results in parts (a)-(c) to describe the difference between eye patterns when there is little distortion in contrast to when there is a lot of distortion.