ECE 405 - REVIEW OF THE BASICS - INVESTIGATION 2
REVIEW OF FOURIER SERIES - PART I

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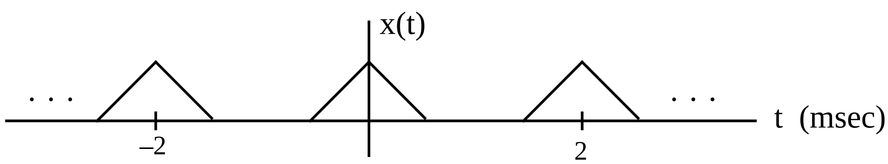
To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

In the last Investigation we reviewed the frequency responses of circuits. In particular we identified passbands and stopbands. The objective of this Investigation is to use Fourier Series to find the spectrums of periodic signals. A very important result is that if the spectrum of a signal is in the passband of a filter than it will reach the output. But if it's in the stopband then it won't.

1. What do we mean when we say a signal $x(t)$ is periodic of period T. How in particular is $x(t+T)$ related to $x(t)$ in a periodic signal of period T

2. Given the following periodic signal

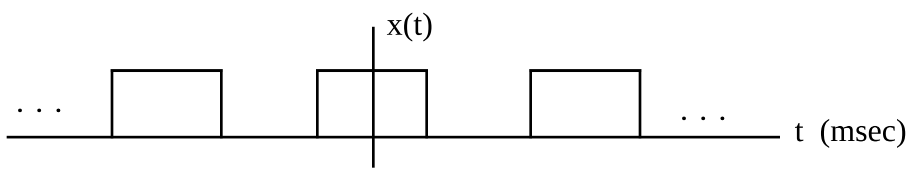


   a. What is the period T in seconds
   b. What is the frequency f in Hz
   c. What is the frequency ω in rad/sec
   d. What is the first harmonic f_o - the fundamental frequency - in Hertz
   e. What is the third harmonic in Hz

3. Sketch a periodic signal with fundamental frequency $f_o = 1\,\text{KHz}$

4. How does increasing the frequency of a periodic signal affect its period

5. Given the following periodic signal



   with Fourier Series expansion as follows

$$x(t) = c_o + \sum_{k=1}^{\infty} c_k \cos(2\pi k f_o t + \theta_k)$$

   Express each of the following signals in terms of x(t) and then sketch it

   a. $x_1(t) = 2c_o + \sum_{k=1}^{\infty} c_k \cos(2\pi k f_o t + \theta_k)$

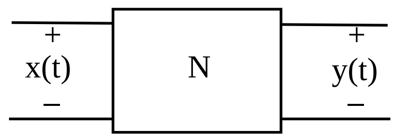
b. $x_2(t) = 2c_o + \sum_{k=1}^{\infty} 2c_k \cos\left(2\pi k f_o t + \theta_k\right)$

c. $x_3(t) = c_o + \sum_{k=1}^{\infty} c_k \cos\left(2\pi k \left(2 f_o\right) t + \theta_k\right)$

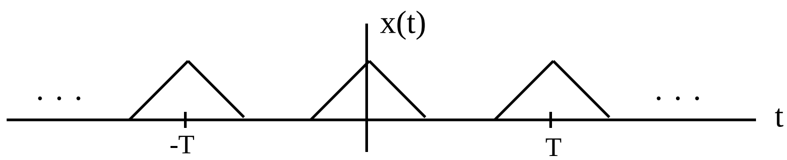
d. $x_4(t) = c_o + \sum_{k=1}^{\infty} c_k \cos\left(2\pi k \left(0.5 f_o\right) t + \theta_k\right)$

e. $x_5(t) = c_o + \sum_{k=1}^{\infty} c_k \cos\left(2\pi k f_o (t + 0.005)\right) + \theta_k\right)$

6. Given the following linear circuit N

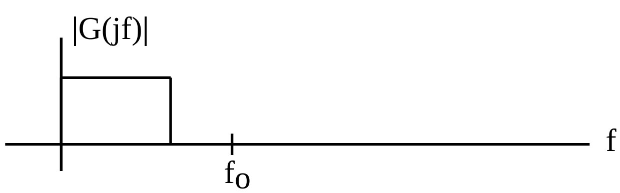


with periodic input x(t) as follows

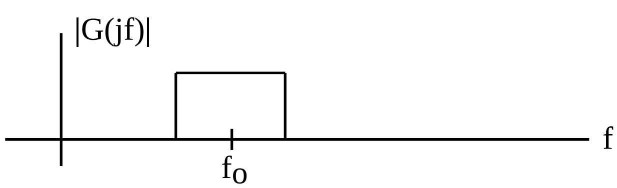


sketch the one-sided spectral plot Y_k and the signal y(t) for each of the following frequency responses

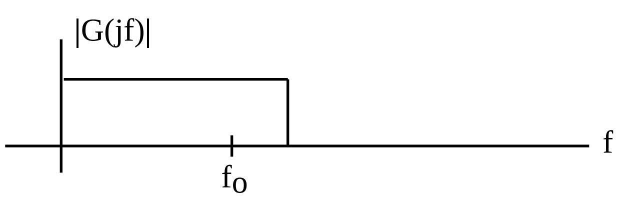
a.



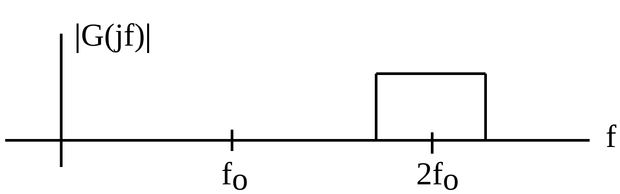
b.



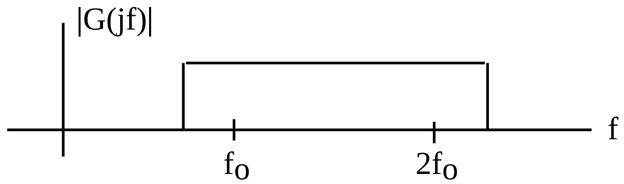
c.



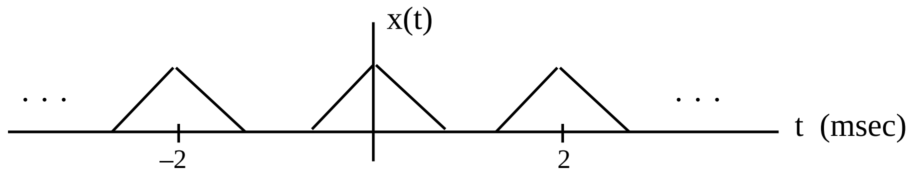
d.



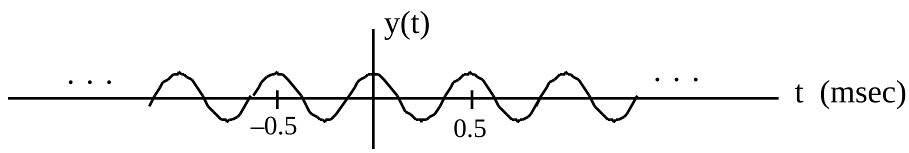
e.



7. Sketch the magnitude of the frequency response of an ideal filter N whose steady state response to the following periodic signal



is



8. Make use of Euler's Relation as follows

$$re^{j\theta} = r\cos(\theta) + jr\sin(\theta) \qquad r\cos(\theta) = \frac{r}{2}e^{-j\theta} + \frac{r}{2}e^{j\theta}$$

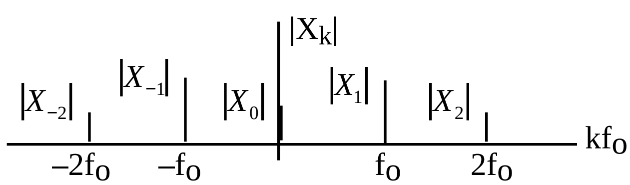
to express the following Fourier Series expansion as a sum of complex exponentials

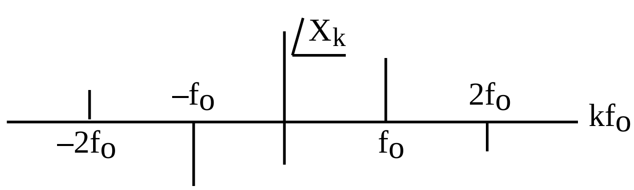
$$x(t) = 3 + 2\cos(2\pi 1000t + 1.2) + 1.5\cos(2\pi 2000t - 1.4)$$

9. Once we've expressed a sum of sinusoids in complex exponential form like we did in Problem (8) as follows

$$x(t) = X_{-2}e^{-j\pi 2000t} + X_{-1}e^{-j\pi 1000t} + X_0 + X_1e^{j\pi 1000t} + X_2e^{j\pi 2000t}$$

we can then graph a double-sided spectral plot of its magnitude and phase like the following





We call this the **spectrum** of x(t). **Memorize** this term. And then graph the double-sided spectral plot of the magnitudes and phases of the harmonics of a periodic signal x(t) with $X_0 = 2$, $X_1 = 3e^{j1.2}$, $X_2 = e^{j}$. Note that $X_{-k} = X_k^*$

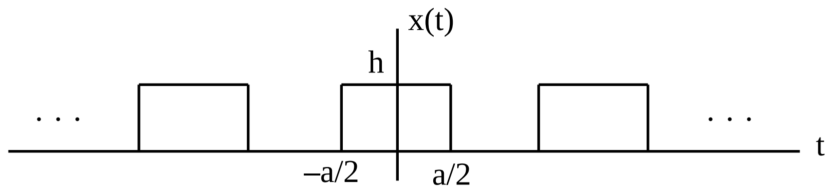
10. Generalizing on the results of Problems (8) and (9) we have

$$c_k \cos(2\pi k f_o t + \theta_k) = \frac{c_k}{2} e^{-j\theta_k} e^{-j2\pi k f_o t} + \frac{c_k}{2} e^{j\theta_k} e^{j2\pi k f_o t} = X_{-k} e^{-j2\pi k f_o t} + X_k e^{j2\pi k f_o t}$$

with $X_{-k} = \frac{c_k}{2} e^{-j\theta_k}$ and $X_k = \frac{c_k}{2} e^{j\theta_k}$ and so we can express a Fourier Series equal to a sum of sinusoids as a sum of complex exponentials as follows

$$x(t) = c_o + \sum_{k=1}^{\infty} c_k \cos\left(2\pi k f_o t + \theta_k\right) = \sum_{k=-\infty}^{\infty} X_k e^{j2\pi k f_o t}$$

In particular it can be shown that the Fourier Series of a pulse train as follows



is a sum of complex exponentials with X_k's given by

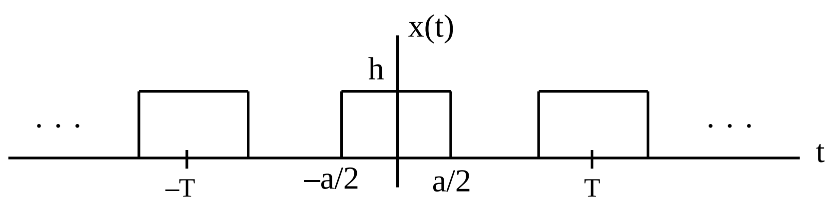
$$X_k = \frac{ha}{T} \frac{\sin(\pi k f_o a)}{\pi k f_o a} = \frac{ha}{T} \operatorname{sinc}(k f_o a) \qquad \text{where} \qquad \operatorname{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

a. Find the constant X_0 and first five harmonics $X_1, \ldots X_5$ of a pulse train with $h = 10$, $a = 0.0002$ and $T = 0.001$
b. Make use of your result in part (a) to find $X_{-1}, \ldots X_{-5}$
c. It's usually easier to graph the spectral plot of a periodic signal like a pulse train by first plotting the **envelope** and then drawing in magnitudes of the harmonics. So draw the envelope for our pulse train as given by

$$X_{env}(f) = \frac{ha}{T} \operatorname{sinc}\left(fa\right)$$

and then draw in the double-sided spectral plot

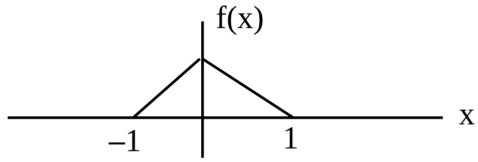
11. Given a pulse train like in Problem (10) as follows



a. How does increasing the magnitude *h* affect the amplitude and zero crossover frequencies of the envelope
b. How does increasing the magnitude *h* affect the spacing and amplitudes of the harmonics
c. How does decreasing the pulse width *a* - without changing *h* or *T* - affect the amplitude and zero crossover frequencies of the envelope
d. How does decreasing the pulse width *a* - without changing *h* or *T* - affect spacing and amplitudes of the harmonics
e. How does increasing the period *T* - without changing *h* or *a* - affect the amplitude and zero crossover frequencies of the envelope

f. How does increasing the period T - without changing h or a - affect the spacing and amplitudes of the harmonics

12. Math Review: Given the following signal



Sketch

a. $2f(x)$
b. $f(x+2)$
c. $f(x-2)$
d. $f(2x)$
e. $f(x/2)$
g. $f(x+2)+f(x)+f(x-2)$