ECE 405 - TRANSITION TO DIGITAL COMMUNICATION - INV 15
INTRODUCTION TO PULSE MODULATION

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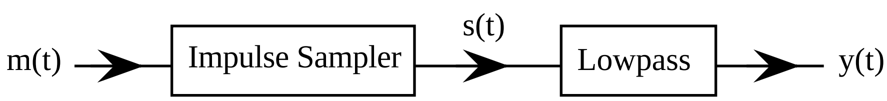
To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

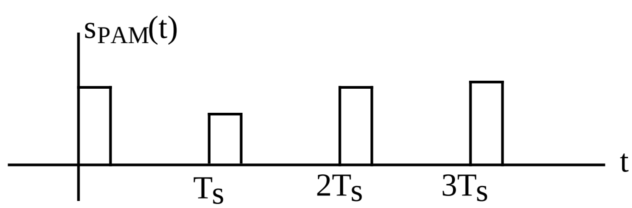
From the last two Investigations we know that in order to be able to reconstruct a bandlimited signal m(t) from its samples we need to sample at a frequency f_s that's more than twice the signal's bandwidth f_b. The objective of this Investigation is to show how the sample values can be transmitted with pulses by varying their amplitudes and duration. As we'll see the results are analogous to those of AM and FM.

1. We begin with a review problem. Given the following circuit



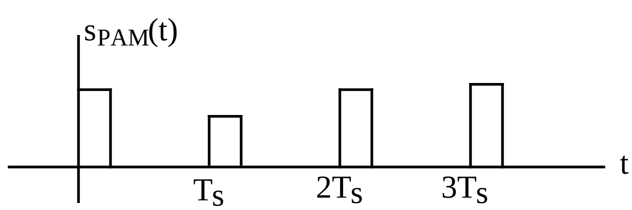
with $m(t) = \cos(2\pi 1000t) + \cos(2\pi 2000t)$ and $f_s = 3500$ samples/sec
   a. Sketch the spectrum $S(f)$
   b. Sketch the spectrum $Y(f)$ if the cutoff frequency of the lowpass filter is $f_c = f_s/2$
   c. Find $y(t)$
   d. How is $y(t)$ related to $m(t)$

2. Once we've sampled a signal m(t) there are a number of ways we can transmit the sample values. One way is to transmit pulses with amplitudes proportional to the amplitudes of the samples as follows

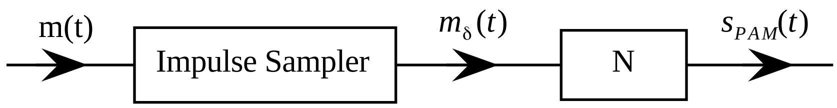


We refer to $s_{PAM}(t)$ as a **Pulse Amplitude Modulation (PAM)** signal
   a. Sketch the PAM signal for $m(t) = 5\cos(2\pi 1000t)$ with $f_s = 3$ KHz
   b. How is PAM similar to AM modulation

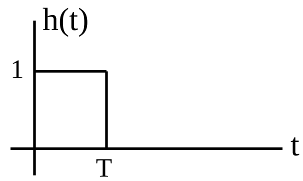
3. One convenient way to obtain the spectrum of a $s_{PAM}(t)$ signal as follows



with pulses of width T is to make use of the fact that $s_{PAM}(t)$ can in principle be obtained as follows



where N is a circuit with impulse response $h(t)$ as follows



Let us first see how this model works. Obtain $s_{PAM}(t)$ for $m(t) = 5\cos(2\pi 1000t)$ with a sampling frequency of $f_s = 5$ KHz as follows

a. First find and sketch $m_\delta(t)$

b. Then make use of your result in part (a) to sketch $s_{PAM}(t)$

4. The objective of this problem is to make use of the model in Problem (3) to find the spectrum of pulse modulated signals

a. First make use of the fact that

$$S_{PAM}(f) = M_\delta(f) F\left[h(t)\right]$$

where $F\left[h(t)\right]$ is the transfer function of N to show that

$$S_{PAM}(f) = T\,\text{sinc}\left(f\,T\right) e^{-j\pi fT} M_\delta(f)$$

b. Now make use of the fact that

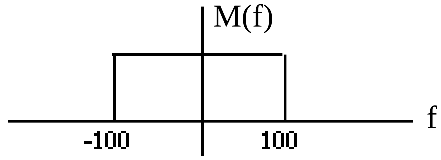
$$M_\delta(f) = F\left[m_\delta(t)\right] = \frac{1}{T_s}\sum_{k=-\infty}^{\infty} M\left(f - kf_s\right)$$

to find $S_{PAM}(f)$

c. Make use of your result in part (b) to sketch the magnitude of $S_{PAM}(f)$ as follows

$$\left|S_{PAM}(f)\right| = \frac{T}{T_s}\left|\text{sinc}\left(fT\right)\right| \sum_{k=-\infty}^{\infty}\left|M\left(f - kf_s\right)\right|$$

if $m(t)$ has the following spectrum, $f_s = 500$ samples/sec and the PAM pulses are of width $T = 1$ msec



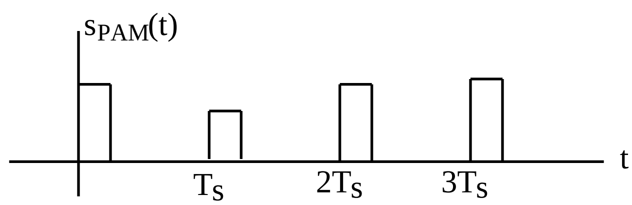
d. Describe how $S_{PAM}(f)$ differs from $M_\delta(f)$

e. Find and sketch the frequency response of the lowpass filter for recovering $m(t)$ from $s_{PAM}(t)$.
f. Describe how your filter's frequency response differs from that of an ideal lowpass.
g. Verify that when

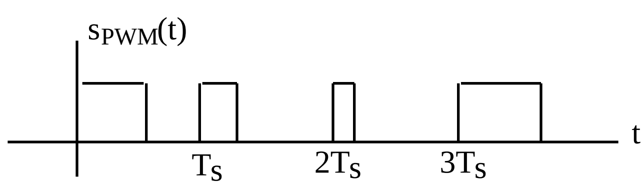
$$\frac{T}{T_s} < 0.1$$

then we should be able get away without a lowpass filter that compensates for the distortion caused by the sinc function

5. Another kind of pulse modulation is **Pulse Width Modulation (PWM)** - also referred to as **Pulse Duration Modulation (PDM)**. In this scheme all the pulses are the same magnitudes but their length varies in proportion to amplitude of the sample. Draw the PWM signal corresponding to the following PAM signal



6. A disadvantage of PWM modulation as introduced in Problem (5) is that we're using a lot of power when the pulses are long. One way to get around this problem is with **Pulse Position Modulation (PPM)** in which just a narrow pulse is transmitted at the end of each PWM pulse.
   a. Sketch the PPM signal corresponding to the following PWM signal



   b. How is PPM modulation similar to phase modulation
   c. Would you expect it to be easier to demodulate PPM or PAM. Why
   d. Would you expect PAM or PPM to be more susceptible to additive noise. Why