## ECE 405 - TRANSITION TO DIGITAL COMMUNICATION - INV 14
## INTRODUCTION TO SAMPLING - PART II

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To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

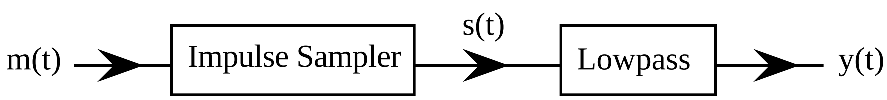
From the last Investigation we know that we can reconstruct a bandlimited signal x(t) from its samples as long as the sampling rate $f_s$ is fast enough to satisfy

$$f_s > 2f_b$$

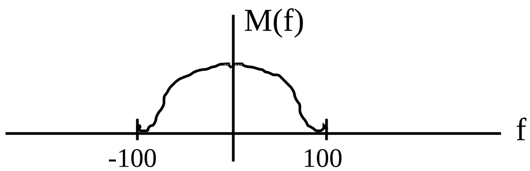
The main objective of this Investigation is to see what happens when we don't sample fast enough.

1. We begin with some review problems. Find and sketch the first five impulses of the impulse sampled signal $x_\delta(t)$ of $x(t) = 5\cos(2\pi 100t + 1.2)$ when $f_s = 300$ samples/sec

2. Given the following system

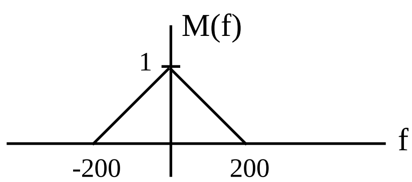



with M(f) equal to the spectrum of the input m(t) as follows




and $f_s$ = 300 samples/sec
   a. Sketch the spectrum S(f)
   b. Sketch the spectrum Y(f) if the cutoff frequency of the lowpass filter is $f_c = 150$ Hz
   c. How is y(t) related to m(t). How can you tell

3. Up to now we've been sampling fast enough to be able to recover bandlimited signals m(t) from their samples. If we don't sample fast enough we get **aliasing** - the distortion caused by not sampling fast enough. **Memorize** this term. Suppose in particular that we sample a signal m(t) with spectrum M(f) as follows




at the sampling frequency $f_S$ = 300 samples/sec
   a. Sketch $M_\delta(f)$
   b. Describe how $M_\delta(f)$ is different from what it would have been if we had sampled faster

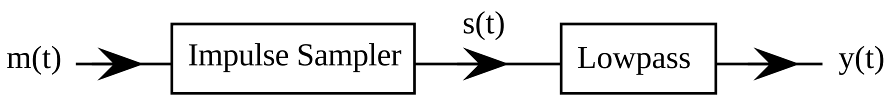
c. Sketch the spectrum of the signal at the output of an ideal lowpass with cutoff frequency

$$f_c = \frac{f_s}{2}$$

d. How do we know that the signal at the output of the lowpass filter is not the original signal $m(t)$

e. Why do we call $f_s/2$ the *folding frequency*

4. The objective of this problem is to see what happens when we don't sample a sum of sinusoids fast enough. Given the following circuit




with $m(t) = \cos(2\pi 1000t) + \cos(2\pi 2000t)$ and $f_s = 2500$ samples/sec

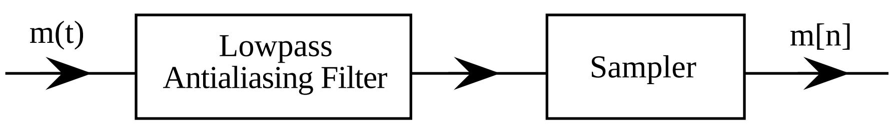
a. Sketch the spectrum $S(f)$

b. Sketch the spectrum $Y(f)$ if the cutoff frequency of the lowpass filter is $f_c = f_s/2$

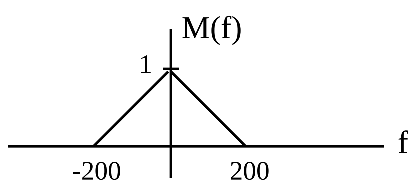
c. Find $y(t)$

d. How is $y(t)$ related to $m(t)$

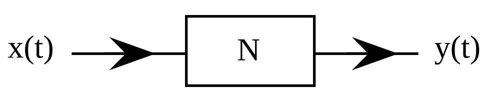
5. In order to avoid aliasing due to high frequency noise or other high frequency interfering signals we typically lowpass filter $m(t)$ before sampling as follows




Draw the frequency response of the anti-aliasing filter if the signal we want to sample has the following spectrum

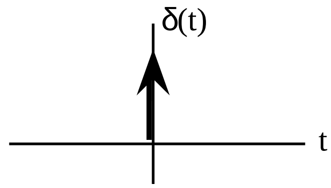



6. The objective of this and the next problem is to review the properties of impulse responses of linear time-invariant circuits for the next Investigation. From circuit analysis we know that the impulse response h(t) of a circuit N as follows



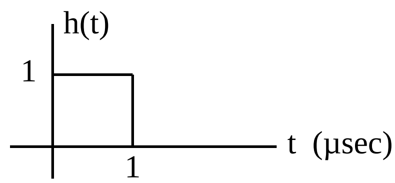

is the response $y(t) = h(t)$ when

(1) The input x(t) is an impulse δ(t) as follows




(2) And all initial conditions are zero

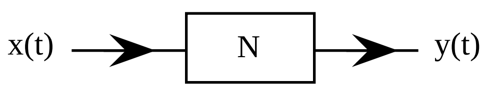
Now suppose the circuit N above has the impulse response h(t) as follows




Then make use of the linearity and time-invariance of N to sketch x(t) and y(t) when

a. $x(t) = 2\delta(t)$
b. $x(t) = 3\delta\left(t - 2\times 10^{-6}\right)$
c. $x(t) = 2\delta(t) + 3\delta\left(t - 2\times 10^{-6}\right)$
d. $x(t) = \delta(t) + 3\delta\left(t - 3\times 10^{-6}\right) + 2\delta\left(t - 6\times 10^{-6}\right)$

7. From Investigation 3 we know that if N is a linear time-invariant circuit as follows




then the Fourier Transform of its output is related to the Fourier Transform of its input as follows

$$Y(f) = G(jf)X(f)$$

and so if $x(t) = \delta(t)$ then $X(f) = 1$ and so

$$Y(f) = G(jf)\ 1 = G(jf)$$

and so the Fourier Transform of the impulse response is equal to the transfer function as follows

$$F[h(t)] = G(jf)$$

Make use of this result to find the frequency response of a circuit with the following impulse response