ECE 405 - ANALOG COMMUNICATIONS - INVESTIGATION 12
INTRODUCTION TO NOISE ANALYSIS

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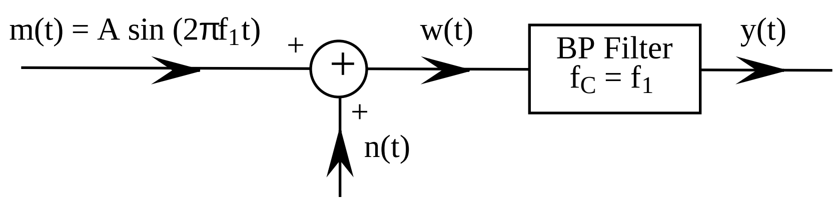
To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

In the last Investigation we introduced power spectral densities for deterministic power signals. The main objective of this Investigation is to extend our results to random signals and then make use of them to compare the performance of AM and FM signals in linear channels with additive Gaussian white noise. Our main result is that FM outperforms AM in the sense that it can trade off wider bandwidth for better noise performance.

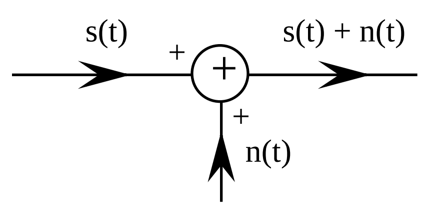
1. We begin by making sketches of what noise looks like. Suppose n(t) is a random noise signal with zero mean
   a. Sketch $n(t)$
   b. Sketch $n(t)$ after it goes through a lowpass filter
   c. Describe the difference between the two signals in parts (a) and (b)

2. Now suppose random noise is added to a sinusoid and then passed through a bandpass filter centered at the frequency f_1 as follows

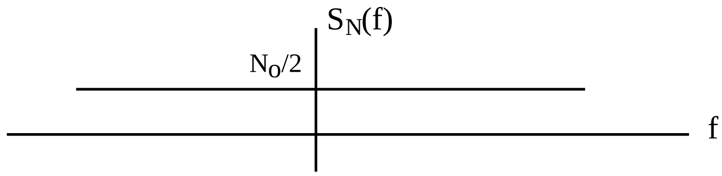


   a. Sketch $w(t)$
   b. Does the noise have more affect on the amplitude of $m(t)$ or the frequency of $m(t)$
   c. Sketch $y(t)$ at the output of the filter
   d. Describe the difference between the way $y(t)$ and $w(t)$ look

3. Explain why AM is more distorted by channel noise than FM is. Hint - make use of the fact that most noise $n(t)$ affecting the transmission of signals through a channel can be modeled as additive noise as follows

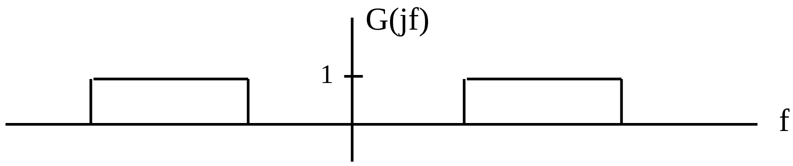


4. From Problem (3) we know that the noise in communication systems is mostly additive. And we know from observation that the amplitudes have a Gaussian distribution. And last but not least we can show from the results of the last Investigation that the power spectral density of channel noise can be modeled as white noise as follows



where N_O is the noise power in Watts/Hertz

a. What's the average power of a white noise signal
b. From part (a) we see that white noise is an idealization - is not real - since its average power is infinite. But this is not really a problem because we only do our calculations at the outputs of bandlimited filters like the following



Sketch the power spectral density $S_O(f) = |G(jf)|^2 S_N(f)$ at the output of the filter

c. Find the average noise power at the output of the filter above if $N_o = 5 \times 10^{-6}$ watts/Hz and the filter bandwidth is $BW = 1$ KHz
d. Find the average noise power if the filter gain is increased to 2

5. The objective of this problem is to get a quantitative measure of how communication systems are affected by random noise. Now from the viewpoint of the end user all that really matters is how much larger the message signal at the output is than is the noise.

   But a high "signal-to-noise ratio" at the output can in general be achieved by simply increasing the power of the transmitted signal. So to get a quantitative measure on how well a given system is "really" doing we calculate its **Figure of Merit** as a ratio of signal-to-noise ratios as follows

$$\text{Figure of Merit } = \frac{(SNR)_o}{(SNR)_c}$$

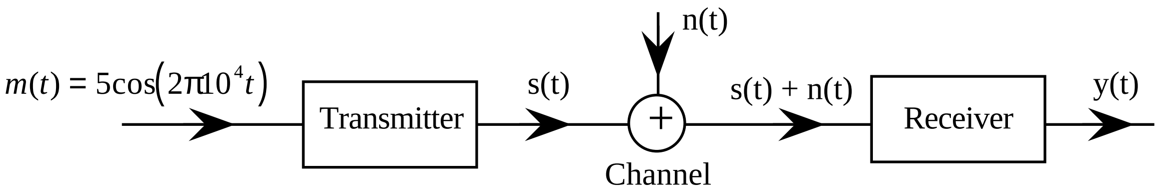
   where

   (1) $(SNR)_o$ is the ratio of the average signal power over the average noise power at the output of the receiver and

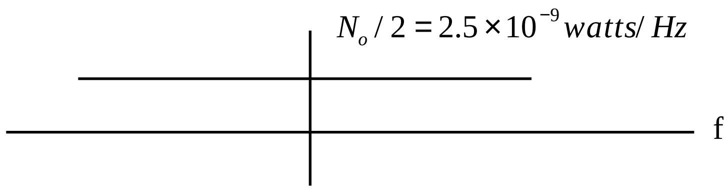
   (2) $(SNR)_c$ is the ratio of the average power of $s(t)$ at the receiver input over the average power of the noise in a bandwidth W where W is equal to the bandwidth of the message signal $m(t)$

   a. Why are Figures of Merit divided by $(SNR)_c$
   b. Would you buy a communication system with a large or small figure of merit. Justify your answer.

6. Given the following transmitter and receiver as follows



with $s(t) =$ modulated output of the transmitter, $n(t) =$ additive white noise of the channel with power spectral density as follows



and the bandwidth W of the message signal is 10 KHz. Find the figure of merit of the receiver if the average power of $s(t)$ is $P_{av} = 5$ mw, the average power of the message signal at the output is $P_{av} = 5$ w and the average power of the noise at the output is $P_{no} = 5$ mw

7. Given that the figure of merit of an AM signal as follows

$$s(t) = A_c\left(1 + k_a m_N(t)\right)\cos\left(2\pi f_c t\right)$$

can be shown to equal

$$\left.\frac{(SNR)_o}{(SNR)_c}\right|_{AM} = \frac{k_a^2 P}{1 + k_a^2 P}$$

where $m_N(t)$ is equal to $m(t)$ scaled to have a maximum value of one and P is the normalized power of the normalized message signal $m_N(t)$

a. Find the figure of merit for the message signal $m(t) = A_m \cos\left(2\pi f_m t\right)$. Note that for this message signal $k_a = A_m / A_c$ and $m_N(t) = \cos\left(2\pi f_m t\right)$
b. How is the figure of merit affected by k_a. Draw a graph to illustrate your result.
c. Explain your result in part (b)
d. What is the figure of merit when $k_a = 1$

8. Given that the figure of merit of an FM signal as follows

$$s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int m(t)dt\right)$$

can be shown to equal

$$\left.\frac{(SNR)_o}{(SNR)_c}\right|_{FM} = \frac{3k_f^2 P}{W^2}$$

where P is the average normalized power of the message signal $m(t)$ and W its bandwidth. How does increasing k_f and therefore the transmission bandwidth B_T affect the figure of merit. Draw a graph to illustrate your result

9. Make use of the result in Problem (8) to verify that the figure of merit of an FM signal with a sinusoidal message signal as follows

$$s(t) = A_c \cos\left(2\pi f_c t + \frac{\Delta f}{f_m}\sin(2\pi f_m t)\right)$$

is equal to

$$\left.\frac{(SNR)_o}{(SNR)_c}\right|_{FM} = \frac{3}{2}\beta^2$$

by first finding $(SNR)_o$ and then $(SNR)_c$

10. From Problems (4) and (7) we see that FM has an advantage over AM because we can increase its figure of merit by increasing its transmission bandwidth B_T while AM is limited to a maximum figure of merit of 1/3 when $m(t)$ is a sinusoid and $k_a = A_m = 1$. What value of β will give us an FM figure of merit equal to 1/3. **Memorize** this result.

11. Find out and describe what the *threshold effect* is