ECE 405 - ANALOG COMMUNICATIONS - INVESTIGATION 10
INTRODUCTION TO PHASE LOCK LOOPS

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To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

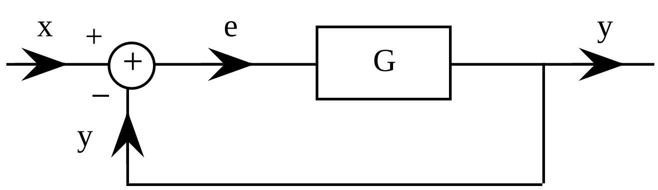
In the last Investigation we showed how FM signals as follows

$$s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int m(t)dt\right)$$

can be indirectly generated with the aid of nonlinear circuit elements and how they can be demodulated with discriminators.

The main objective of this Investigation is to introduce phase lock loops (PLL) and show how they can be used to demodulate FM signals. Now the general analysis of phase lock loops is fairly complicated because they're nonlinear feedback circuits. But once they're in *lock*, as they are in this Investigation, we can approximate them by linear circuits.

1. We begin with a review of some basic results of linear feedback systems with unity feedback as follows

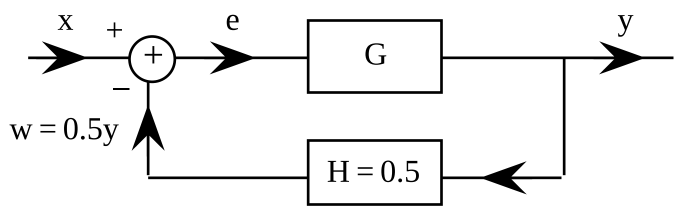


   a. Find y and e as a function of x when $G = 1$
   b. Find y and e as a function of x when $G = 2$
   c. Find y and e as a function of x when $G = 10$
   d. Make use of your results in parts (a)-(c) to sketch y as a function of G. Describe what's happening to y as G increases
   e. Make use of your results in parts (a)-(c) to sketch e as a function of G.
   f. Describe what's happening to e - and therefore the relationship between x and y - as G increases
   g. Explain in words why y and e behave the way they do as G increases

2. Given the linear feedback circuit of Problem (1) with x=10 and $G=10^5$
   a. Find y
   b. Find y if G decreases by 50%
   c. Find y if G increases by 50%
   d. By how much did y change in parts (b) and (c) as the value of G changed
   e. What's the significance of your result in part (d) - how critical is the value of G when G is "large"

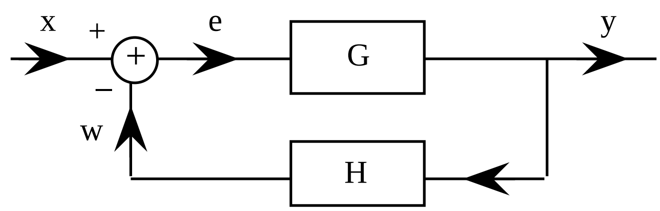
3. From Problem (2) we see that as long as the loop gain G of the unity gain feedback circuit is large then the output will depend only on the input. In particular it doesn't matter "how large" G is as long as it's "large". The objective of this problem is to see what happens in a linear

feedback circuit when the feedback is different from unity as follows

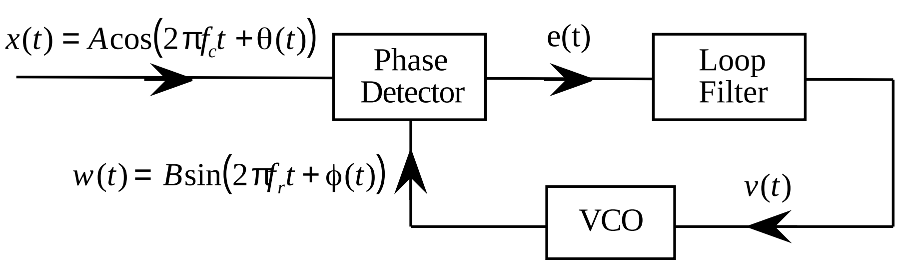


a. Find y and e as a function of x when $G = 1$
b. Find y and e as a function of x when $G = 2$
c. Find y and e as a function of x when $G = 10$
d. Find y and e as a function of x when $G = 100$
e. Make use of your results in parts (a)-(d) to sketch y as a function of G. Describe what's happening to y as G increases
f. Make use of your results in parts (a)-(d) to sketch e as a function of G. Describe what's happening to e as G increases
g. Explain in words how the feedback $H = 0.5$ is affecting y

4. From our review of "regular" feedback systems as follows



we see that as long as the loop gain is large then $w \approx x$ and so the output y will be a multiple of x that depends on the value of H. The objective of this problem is to introduce **phase lock loops** as follows



The basic components of phase lock loops consist of a phase detector, loop filter and VCO as follows

(1) The phase detector is a multiplier followed by a lowpass filter with output $e(t)$

(2) The loop filter is a filter with large gain that controls the dynamic behavior of the feedback loop

(3) The VCO is a voltage controlled oscillator with output $w(t)$ equal to a sinusoid with instantaneous frequency

$$f_i(t) = f_r + k_{vco} v(t)$$

a. Why is f_r called the "free running" frequency of the VCO
b. Describe how $v(t)$ affects the instantaneous frequency $f_i(t)$
c. Make use of the fact that $w(t)$ is of the form

$$w(t) = B\sin\left(2\pi f_r t + \phi(t)\right)$$

to obtain an expression for its instantaneous frequency $f_i(t)$ in terms of f_r and $\phi(t)$

d. Make use of your result from part (c) as follows

$$f_i(t) = \frac{1}{2\pi}\frac{d}{dt}\left(2\pi f_r t + \phi(t)\right) = f_r + \frac{1}{2\pi}\frac{d}{dt}\phi(t) = f_r + k_{vco}v(t)$$

to find $w(t)$ if $B = 5$, $f_r = 100MHz$, $k_{vco} = 10^7$ and $v(t) = 5\cos\left(2\pi 10^4 t\right)$

e. Make use of the equation in part (d) to express $v(t)$ in terms of $\phi(t)$
f. Find $v(t)$ if $k_{vco} = 10^4$ and $\phi(t) = 20\sin(2\pi 10^4 t)$

5. From Problem (4) we know that phase lock loops work basically the same as "regular" feedback circuits except that when they're in *lock* with $e(t) \approx 0$ it's the phases of $x(t)$ and $w(t)$ - not their amplitudes - that are equal with $f_r = f_c$ and $\phi(t) \approx \theta(t)$. Therefore if a phase lock loop is in lock

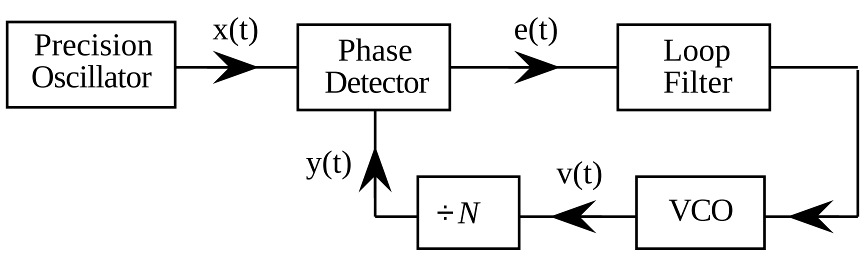
$$v(t) = \frac{1}{2\pi k_{vco}}\frac{d}{dt}\phi(t) = \frac{1}{2\pi k_{vco}}\frac{d}{dt}\theta(t)$$

Make use of this result to show how an FM signal with

$$x(t) = A_c\cos\left(2\pi f_c t + 2\pi k_f \int m(t)dt\right)$$

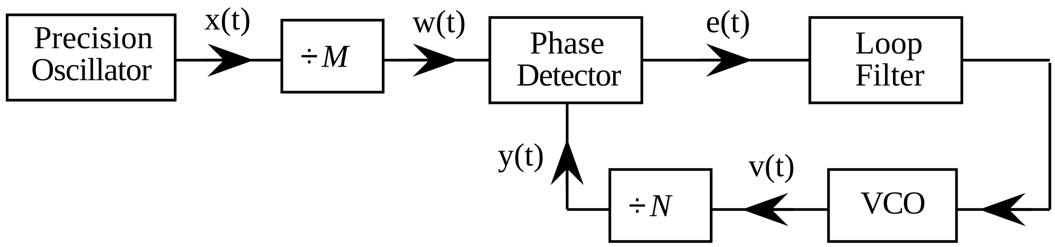
can be demodulated

6. The objective of this and the next problem is to show how phase lock loops can be used to generate sinusoids with frequencies equal to multiples of an input sinusoid. Show how the following phase lock loop circuit



with a divide by N counter that frequency divides the output of the VCO by a factor of N is able to generate a sinusoid v(t) with a frequency $f_v = Nf_x$ equal to N times that of the precision oscillator.

7. Find the frequency f_v of v(t) in the following circuit in terms of the frequency f_o of the oscillator. Again note that it's the frequencies of the signals $x(t)$ and $v(t)$ that are getting divided by M and N and not their amplitudes.


Precision Oscillator
x(t)
÷M
w(t)
Phase Detector
e(t)
Loop Filter
y(t)
÷N
v(t)
VCO