ECE 315 - DISCRETE RANDOM VARIABLES - INVEST 5
INTRODUCTION TO DISCRETE RANDOM VARIABLES

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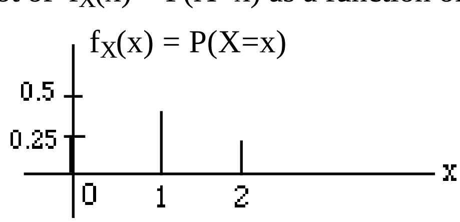
To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

From the first four Investigations we know how to calculate the probabilities and conditional probabilities of events. The objective of this Investigate is to introduce random variables - functions that assign numbers to the outcomes of our random experiments. Suppose, for example, that we flip a coin three times with sample space S = {TTT, TTH, THT, . . . }. One example of a random variable is the function that assigns to each outcome the number of heads. Random variables are important because they enable us to easily specify events, calculate things like averages and draw graphs.

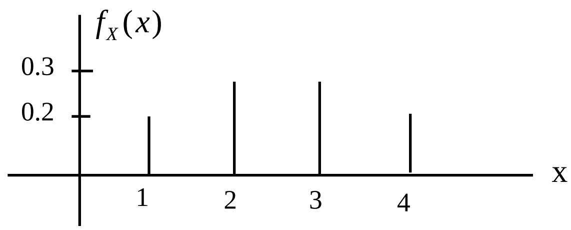
1. As we said in the introduction, **random variables** are functions that assign numbers to the outcomes of our random experiments. **Memorize** this definition. And then make up two random variables X and Y for when we flip a coin three times.

2. Let us now flip a fair coin three times with **random variable** X equal to the number of heads
   a. What are the values of the random variable X
   b. Set up a Table containing each possible outcome and its value of X
   c. Draw a Venn diagram with four events - one for each value of X
   d. As we can see from the Venn diagram in part (c), a random variable defines a partition of the sample space. What are the outcomes in each of the events making up the partition for the random variable X of this problem.

3. Once we have a random variable X the next step is to calculate the probabilities of its different values. Suppose, in particular, that X is equal to the number of heads when we flip a fair coin three times like in Problem (1)
   a. Find the probabilities for the different values of X - the probabilities P(X=0), P(X=1), P(X=2) and P(X=3). Put your results in a Table.
   b. Then make a discrete plot of $f_X(x) = P(X=x)$ as a function of x like the following



4. The function $f_X(x) = P(X = x)$ we calculated in Problem (3) is an example of a **probability distribution function**. It tells us the probability that the random variable X will equal the value x when we do the random experiment. Given the following probability distribution function



Find the following probabilities

a. $P(1 < x \le 3)$
b. $P(1 \le x \le 3)$
c. $P(x > 1)$
d. $P(x \ge 1)$

5. Find and then make a discrete plot of the probability distribution function $f_X(x)$ for the number of defective computers in a shipment of 2 computers randomly chosen from a group of 5 computers, 3 of which were defective. Hint - find the probability by listing all the possible outcomes with W_1 and W_2 the working computers and D_1, D_2 and D_3 the defective ones.

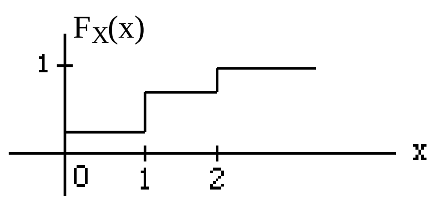
6. In Problem (3) we calculated the probabilities of the random variable X equal to the number of heads when we flip a fair coin three times. More generally we're interested in the probability that the value of X is in a given interval like $1 < x \le 3$. Make use of your results in Problem (2) to find the probability $P(1 < x \le 3)$. Describe how you got your result.

7. Generalizing on the result of Problem (5) we find that it's usually easier to calculate the probability that a random variable X is in a given interval as follows

$$a < X \le b$$

if we first calculate the **cumulative distribution function $F_X(x) = P(X \le x)$**. Suppose, as in Problem (5) that X is the number of heads when we flip a fair coin three times.

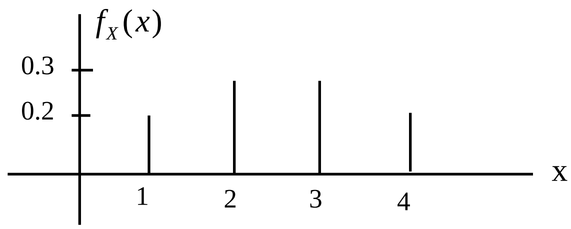
a. Find the cumulative probabilities $F_X(x) = P(X \le x)$ for $x = 0, 1, 2, 3$. Explain how you got your answers. Put your results in a Table.
b. Make a plot of $F_X(x)$ like the following



as a function of the continuous variable x.

c. Now make use of your $F_X(x)$ to find $P(1 < x \le 3) = F_X(3) - F_X(1)$
d. Explain in words why $P(1 < x \le 3) = F_X(3) - F_X(1)$

8. Sketch $F_X(x)$ for the following probability distribution



9. The objective of this problem is to do another example involving cumulative distribution functions $F_X(x)$. Suppose X is a random variable with a probability distribution function $f_X(x)$ as follows

x	$f_X(x)$
0	0.1
1	0.2
2	0.4
3	0.1
4	0.2

a. Find the probability $P(1 < x \le 3)$. Explain how you got your result
b. Calculate the cumulative distributive function $F_X(x)$ and add the values to your Table
c. Plot $f_X(x)$ and $F_X(x)$ versus x
d. Verify and then explain why the result you got in part (a) is equal to the following function of $F_X(x)$

$$P(1 < x \le 3) = F_X(3) - F_X(1)$$

e. Generalizing on the results in part (d) we have

$$P(a < x \le b) = F_X(b) - F_X(a)$$

Explain in words why $F_X(x)$ is a nondecreasing function starting at 0 and ending at 1. **Memorize** this result.

10. The objective of this problem is to introduce conditional probabity distribution functions $f_{X|B}(x|B) = P(X=x|B)$. Once again let's suppose that X is the number of heads when we flip a coin three times and that B is the event that the first flip is a head. Find these probabilities and put them in a Table.

11. Generalizing on the conditional probability distribution function of Problem (10) we now define $f_{X|Y}(x|y) = P(X=x|Y=y)$. Calculate $f_{X|Y}(x|y)$ and put the results in a Table if X again equals the number of heads when we flip a fair coin three times and $Y = 0$ when the first flip is a head and $Y = 1$ when the first flip is a tail.