ECE 315 - BASIC PROBABILITY - INVESTIGATION 3
INDEPENDENCE OF DISCRETE EVENTS

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To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

From the first two Investigations we know that

$P(A \cup B)$ = Fraction of time the result of a random experiment is in A or B
$P(A \cap B)$ = Fraction of time the result of a random experiment is in both A and B
$P(A \mid B)$ = Fraction of time the result of a random experiment is in A when it is in B

when we do the random experiment a whole bunch of times. And from the last Investigation we know that adding a condition B can affect the probability of A in any of the following ways

$$P(A \mid B) < P(A) \qquad P(A \mid B) = P(A) \qquad P(A \mid B) > P(A)$$

depending on the events A and B. The objective of this investigation is to focus in on the special case of

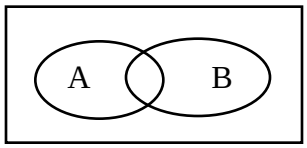
$$P(A \mid B) = P(A) \quad \text{or equivalently} \quad \frac{n_{A \cap B}}{n_B} = \frac{n_A}{n}$$

In this case we say that A is independent of B.

1. The objective of this first problem is to get a handle on what is meant by **independent events**. Suppose a group of doctors is testing a new drug on a group of sick people - randomly giving some of them a new drug and the rest a placebo. Now with

   A = Patients getting better B = Patients taking the new drug

   as indicated in the following Venn diagram

   which of the following relations hold

   $$P(A \mid B) < P(A) \qquad P(A \mid B) = P(A) \qquad P(A \mid B) > P(A)$$

   a. When the new drug is making things better - explain in words how you know
   b. When the new drug is making things worse - explain in words how you know
   c. When the new drug is having no affect - explain in words how you know
   d. In which case would you say that the patient's getting better is independent of the new drug. Why

2. From Problem (1c) we saw that if the drug has no affect - if your chances of getting better are **not affected** by whether or not you're in the group taking the drug - then

$$P(A \mid B) = P(A) \quad \text{or equivalently} \quad \frac{n_{A \cap B}}{n_B} = \frac{n_A}{n}$$

and we say A is **independent** of B. Now let's suppose we flip a coin two times

a. Find two events A and B that are indpendent
b. Show that your events A and B are independent - that they satisfy $P(A \mid B) = P(A)$
c. Find two events C and D that are not independent
d. Show that your events C and D are not independent - that $P(C \mid D) \neq P(C)$

3. Explain in your own words what it means for two events A and B to be indpendent

4. The objective of this problem is to generate data for events that are independent and events that are not independent. Suppose we do a random experiment n = 1000 times. Come up with values for

n_A = Number of times the outcome of the random experiment is in A
n_B = Number of times the outcome of the random experiment is in B
$n_{A \cap B}$ = Number of times the outcome of the random experiment is in both A and B

a. If A is independent of B. Verify your values
b. If A is not independent of B. Verify your values

5. Now let's draw some pictures. Suppose $P(A) = P(B) = 0.5$
a. Draw a Venn Diagram for A and B if A is independent of B. Explain how it can be seen from your Venn Diagram that A and B are independent
b. Draw a Venn Diagram for A and B if A is not independent of B. Explain how it can be seen from your Venn Diagram that A and B are not independent

6. Show and then **memorize** the fact that if A and B are independent then

$$P(A \cap B) = P(A)P(B)$$

Hint - make use of the fact that

$$P(A \cap B) = P(A \mid B)P(B)$$

Note that mathematicians usually define independence by the relationship $P(A \cap B) = P(A) P(B)$ and then derive from it the result that independent events satisfy $P(A \mid B) = P(A)$

7. Now suppose we have a sample space S with equally likely outcomes $S = \{w, x, y, z\}$. Make use of the result from the previous problem that two events A and B are independent if they satisfy

$$P(A \cap B) = P(A)\, P(B)$$

to determine which, if any, of the following pairs of events are independent. Illustrate your results with Venn Diagrams

a. $A = \{w, x, y\}$ and $B = \{y, z\}$
b. $C = \{x, y\}$ and $B = \{y, z\}$

8. Do the calculations for a coin flipping experiment to illustrate the fact that if A is independent of B with $P(A \mid B) = P(A)$ then B is independent of A with $P(B \mid A) = P(B)$

9. Generalize on your example in Problem (7) to show that whenever A is independent of B then B is independent of A. Make use of the fact that

$$\frac{n_{A\cap B}}{n_B} = \frac{n_A}{n} \qquad \frac{n_{B\cap A}}{n_A} = \frac{n_B}{n}$$

10. Explain in words why two mutually exclusive events A and B cannot be independent - that is, be independent and satisfy $A \cap B = \varnothing$. Illustrate your result with an example

11. We know from the problems we've been doing in this investigation that two events A and B are independent if

$$P(A|B) = P(A) \quad \text{or equivalently} \quad P(A \cap B) = P(A)P(B)$$

Now suppose we're flipping two coins. Describe in words what we mean when we say the two coins are independent

12. Suppose we flip two independent coins that each have a probability of heads $P_H = 0.6$
    a. List all possible outcomes
    b. Find the probability of each possible outcome. Hint - make use of the result for independent events that $P(A \cap B) = P(A)P(B)$
    c. Find the probability of at least one head

13. Determine if the following two events are independent when we flip two fair and independent coins

A = {First coin a head} B = {At least one head}