ECE 315 - INTRODUCTION TO STATISTICS - INVEST 23
HYPOTHESIS TESTING

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To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

In the last Investigation we introduced confidence intervals in order to get a handle on how close estimated means are to real means. The objective of this Investigation is to introduce hypothesis testing for determining how likely it is that the real mean μ of a population is different from a given value μ_o we refer to as the null hypothesis.

1. We begin with a review problem. Find the 0.01 confidence interval of the mean for *n* samples with $\overline{X} = 20$ and $s = 2$ when the sample size is
   a. n = 100
   b. n = 20

2. The objective of this problem is to introduce hypothesis testing with an example. Suppose a semiconductor company needs an average success rate of 21 working chips per wafer for a new process to be profitable. In order to decide whether to use the new process or not the company therefore has to decide between the following two hypotheses

   Null Hypothesis H_0: $\mu = \mu_0 = 20$ chips/wafer

   Alternative Hypothesis H_1: $\mu > 20$ chips/wafer

   To determine which hypothesis is most likely true we do the following *hypothesis test*

   (1) Take a random sample of N wafers
   (2) Calculate the average yield $\overline{X}$ of the N samples
   (3) Make use of the sample variance s^2 (or some other means of estimating s) and the fact that $\overline{X}$ is Gaussian (as long as N ≥ 30) to find the probability that $\overline{X}$ is a sample from a population with mean $\mu = \mu_o$

   Now since semiconductor fabrication facilities are very expensive we only want to make the investment - reject the null hypothesis - if we're pretty sure that the new chip process really is profitable. Only if the **significance level** - probability that we're wrong - is something relatively small like 0.01.

   Given all this suppose we test 30 wafers and obtain 10 each with yields 20, 22 and 24
   a. Find the sample mean and sample variance
   b. Make use of the sample variance to sketch what the probability density would look like if the population mean was μ_o
   c. Make use of your probability density in part (b) to find the probability that the sample mean $\overline{X}$ is from a population with mean μ_o
   d. Would you accept or reject the null hypothesis if you wanted to be wrong at most 1 time in 100.

3. The objective of this problem is to calculate the probability of making an error when we do hypothesis testing. Suppose in particular we have a problem with null and alternative

hypotheses as follows

Null Hypothesis H_0 : $\mu = \mu_0 = 50$ chips/wafer

Alternative Hypothesis H_1 : $\mu < 50$ chips/wafer

And a sample with $\bar{X} = 46$ and $\sigma_{\bar{X}} = 1.5$. Then we can make two kinds of errors as follows

(1) **Type I error** - Incorrectly reject the null hypothesis
(2) **Type II error** - Incorrectly reject the alternative hypothesis

Memorize these terms. Then assuming we reject the null hypothesis whenever $\bar{X} \le 47$ - which we refer to as the **cutoff** or **critical point**

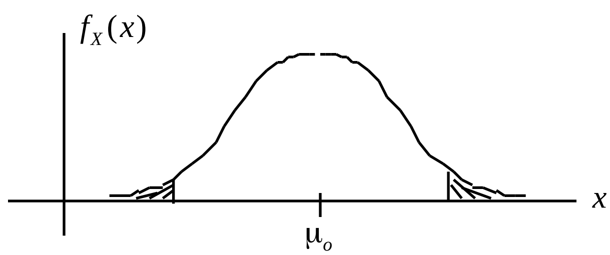
a. Find the probability α of a Type I error. Illustrtate with a sketch of the probability density.
b. Find the probability β of a Type II error. Ilustrate with a sketch of the probability density.

4. Up to now our hypothesis tests have been **single-tailed**. We've been testing whether $\mu > \mu_o$ or $\mu < \mu_o$. In **two-tailed** tests as follows

Null Hypothesis H_0 : $\mu = \mu_0$

Alternative Hypothesis H_1 : $\mu \neq \mu_o$

we're testing to see if the population mean is near μ_o or is some value greater or less than μ_o as follows



where each of the shaded regions has a probability of $\alpha/2$. What are the cutoff points if $\mu_o = 100$, $\sigma_{\bar{X}} = 5$ and $\alpha = 0.01$