ECE 315 - BASIC PROBABILITY - INVESTIGATION 2
DISCRETE CONDITIONAL PROBABILITIES

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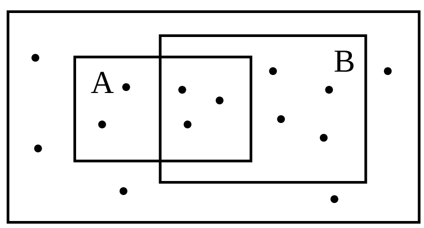
To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

From the first investigation we know that the **probability** P(A) of an event A is as follows

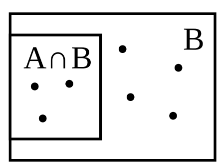
$$P(A) = \frac{n_A}{n} = \text{ Fraction of time the result of a random experiment is an outcome in A}$$

when we do the random experiment a **whole bunch** of times n. The objective of this investigation is to calculate what we refer to as conditional probabilities.

1. The goal of this and the next two problems is to introduce the idea of conditional probabilities with some examples. Suppose we do a random experiment a "whole bunch" of times - putting dots in the corresponding regions as follows



   a. Find P(A)
   b. Now suppose we look at just those outcomes from our experiment that turn up in B as follows



   Now what's the probability that the outcome will be in A - the probability that the outcome of the random experiment is in A when we restrict ourselves to just looking at those outcomes from the random experiment that are in B

2. Suppose we have a sample space S with equally likely outcomes

$$S = \{w, x, y, z\}$$

   a. What is the probability that the outcome of a random experiment is in A = {w, x, y}
   b. What is the probability that the outcome of this random experiment is in A if somehow or other we've been able to find out that it's in B = {y, z}. Draw a Venn Diagram to illustrate what's going on

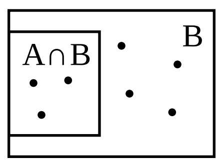
3. Now suppose we flip three coins with all possible outcomes equally likely
   a. Make a Table of all the possible outcomes
   b. What's the probability of at least two heads

c. What's the probability of at least two heads altogether if the first coin is a head. Draw a Venn Diagram to illustrate what's going on
d. How did adding the condition in part (c) affect the probability of at least two heads altogether
e. What's the probability of at least two heads altogether if the first flip is a tail. Draw a Venn Diagram to illustrate what's going on
f. How did adding the condition in part (e) affect the probability of getting at least two heads altogether
g. Now flip three coins 50 times and then put the following information in a Table: the fraction of time you got at least two heads; the fraction of time you got at least two heads when the first flip was a head; and the fraction of time you got at least two heads when the first flip was a tail
h. Compare you experimental results in part (g) with your calculated results above. Be sure to include percentage differences. Put your results in a Table

4. The probabilities we just calculated in Problem (1b), Problem (2b), and Problems (3c) and (3e) are called conditional probabilities. We denote conditional probabilities as follows

P(A|B) = Conditional Probability of A given B

with P(A|B) equal to the probability that the outcome of a random experiment is in A when we restrict ourselves to just looking at those outcomes in B as illustrated by our Venn Diagram from Problem (1) as follows

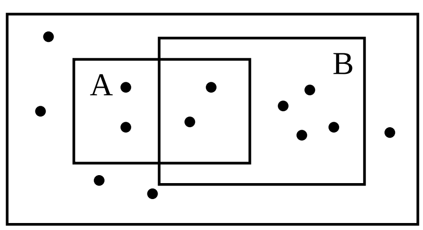


The conditional probability P(A|B) is given by the fraction

$$P(A|B) = \frac{n_{A \cap B}}{n_B} = \frac{\text{Number of times the outcome is in A and B}}{\text{Number of times the outcome is in B}}$$

when we do the random experiment a "whole bunch" of times

a. How is what we mean by P(A|B) similar to what we mean by P(A).
b. Find P(A) and P(A|B) given the following



c. Find P(A|B) for A = {w, x} and B = {x, y} if the results of doing the random experiment a whole bunch of times are as follows: w, w, y, z, x, w, z, y, x, x, w

5. Making use of the result from Problem (4) we have that

$$P(A \mid B) = \frac{n_{A \cap B}}{n_B} = \frac{n_{A \cap B}/n}{n_B/n} = \frac{P(A \cap B)}{P(B)}$$

Memorize this result. Then verify that it gives the same answers as you got in

a. Problem 1(b)
b. Problem 2(b)
c. Problem 3(c)
d. Problem 3(e)

6. The objective of this and the rest of the problems of this investigation is to look at some of the basic properties of conditional probabilities. Draw Venn Diagrams with dots for the results of the random experiments that **clearly** illustrate that each of the following is possible

   a. $P(A_1|B) < P(A_1)$ b. $P(A_2|B) = P(A_2)$ c. $P(A_3|B) > P(A_3)$

7. Now suppose we again consider the sample space of Problem (1) with equally likely outcomes S = {w, x, y, z}. Let B = {w, x}. Find events A_1, A_2 and A_3 such that

   a. $P(A_1|B) < P(A_1)$ b. $P(A_2|B) = P(A_2)$ c. $P(A_3|B) > P(A_3)$

   Calculate the probabilities

8. Now let's suppose we flip a fair coin two times. Let B be the event of exactly one head.
   a. Verify that if A_1 is the event no heads then $P(A_1|B) < P(A_1)$
   b. Verify that if A_2 is the event the first flip is a tail then $P(A_2|B) = P(A_2)$
   c. Verify that if A_3 is the event at least one head then $P(A_3|B) > P(A_3)$

9. Solving for $P(A \cap B)$ in our result

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

   we have

$$P(A \cap B) = P(A \mid B)P(B)$$

   Memorize this result. Then
   a. Describe in words what it says.
   b. Verify it's true when A = {at least one head} and B = {at least one tail} when we flip a coin twice

10. Suppose we do a random experiment a whole bunch of times and find that 1/3 of the time the result is in B and 1/4 of the time that it's in B it's also in A. Then what's the probability that the result of the random experiment is in both A and B.

11. What does $P(B|A) = 0$ imply about the outcomes in A and B. Illustrate what's going on with a Venn diagram.