ECE 315 - CONTINUOUS RANDOM VARIABLES - INVEST 16
THE GAUSSIAN DISTRIBUTION - PART II

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To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

In the last Investigation we introduced the normal or Gaussian distribution with probability density function as follows

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

We showed in particular that

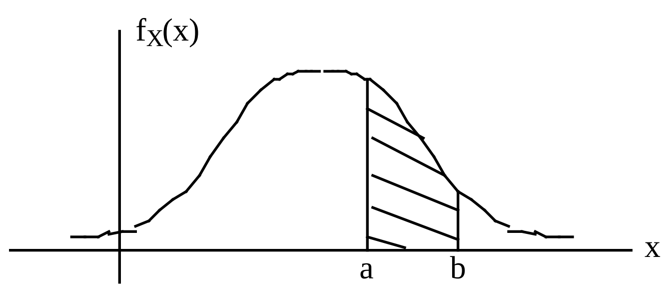
$$F_X(b) - F_X(a) = F_Z\left(\frac{b-\mu}{\sigma}\right) - F_Z\left(\frac{a-\mu}{\sigma}\right) = \int_{(a-\mu)/\sigma}^{(b-\mu)/\sigma} \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\, dz = \int_{(a-\mu)/\sigma}^{(b-\mu)/\sigma} f_z(z)\, dz$$

where

$$f_Z(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$

is the normalized Gaussian probability density function with $\mu = 0$ and $\sigma = 1$. The objective of this investigation is to get more experience with the Gaussian distribution and show how it can be used to approximate the binomial distribution.

1. Given the following graph of a Gaussian distribution with parameters μ and σ



   a. Sketch $F_X(x)$
   b. Find $F_X(\mu)$
   c. Shade in the corresponding region on a normalized Gaussian distribution

2. Express the following probabilities in terms of the normalized Gaussian distribution $F_Z(z)$

   a. $P(\mu - \sigma \le X \le \mu + \sigma)$
   b. $P(\mu - 2\sigma \le X \le \mu + 2\sigma)$
   c. $P(\mu - 3\sigma \le X \le \mu + 3\sigma)$

3. Make use of a Table of the normalized Gaussian distribution (or your calculator) to find the

probabilities in Problem (2). Then in each case draw graphs of the Gaussian densities that indicate the corresponding regions.

4. Suppose that the values of a batch of 1K resistors has a normal distribution with mean $\mu = 1K$ and standard deviation $\sigma = 25\,\Omega$. What's the probability that any given resistor is within 5% of 1K. Hint - start by sketching a graph of $f_X(x)$

5. Suppose 99% of a batch of 1K resistors with a normal Gaussian distribution are within 1% of 1K. What is the standard deviation σ of the distribution

6. We can, of course, only approximate any real probability distribution as Gaussian. What, in particular, does the Gaussian distribution predict for our resistors that is not physically possible

7. As we would expect from the last Investigation the Gaussian distribution can be used to approximate the binomial distribution when n is large. A good rule of thumb is that such approximations are reasonable if $npq > 10$. Make use of this fact to find the probability of at least 915 success in $n = 1000$ trials of a binomial distribution with $p = 0.9$. Remember that for binomial distributions $\mu = np$ and $\sigma^2 = npq$

8. Suppose we take a sample of 1000 beer cans to see how many are not filled to the top. What's the probability that the sample will contain more than 50 partially filled cans if the probability of any given can not being filled to the top is $p = 0.03$. Assume independence among the cans

9. The objective of this problem is to introduce some common notation. Make use of the following normalized equation for $F_X(b) - F_X(a)$

$$F_X(b) - F_X(a) = \int_{(a-\mu)/\sigma}^{(b-\mu)/\sigma} \frac{1}{\sqrt{2\pi}}\, e^{-z^2/2}\, dz$$

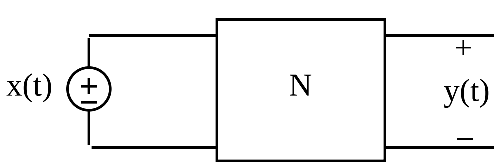
to derive and find the following relations. As usual graphs may be helpful

   a. Show that $F_X(b) - F_X(a) = \operatorname{erf}\left(\frac{b-\mu}{\sigma}\right) - \operatorname{erf}\left(\frac{a-\mu}{\sigma}\right)$ were $\operatorname{erf}(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-u^2/2}\, du$

   b. Find $F_X(b) - F_X(a)$ as a function of $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-u^2/2}\, du$

   c. Find $F_X(b) - F_X(a)$ as a function of $Q(x) = \operatorname{erfc}(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-u^2/2}\, du$

10. Review Problem: Given the following circuit



   a. Sketch |G(j2πf)| if N is a lowpass filter. Then explain why it's called lowpass
   b. Sketch |G(j2πf)| if N is a highpass filter. Then explain why it's called highpass
   c. Sketch |G(j2πf)| if N is a bandpass filter. Then explain why it's called bandpass