ECE 315 - BASIC PROBABILITY - INVESTIGATION 1
INTRODUCTION TO DISCRETE PROBABILITY

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To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

The objective of this first investigation is to "define" what we mean by probability and explore some of its basic properties.

1. There are many situations like the flipping of a coin where we know the possible **outcomes** - we know that the coin will come up heads or tails - but we don't know what the result will be for any given flip until we actually do it. Despite this we can still use a number of methods to estimate the **chances** - the **probability** - that the coin will come up heads or tails. One way to get these probabilities is to make educated guesses based on the geometry (symmetry) of a given coin. Alternatively we can simply flip the coin a "whole bunch" of times and see what happens. This is referred to as the **relative frequency** method of defining probabilities.
   a. Flip a coin a "bunch" of times (like around 50). Put your results in a Table.
   b. What do your results in part (a) predict is the probability of your coin coming up heads - the **fraction** of time you would expect the coin to come up heads if you flipped it a **whole bunch** of times.
   c. Do you think your coin is **fair** - equally likely to come up heads as tails. Why

2. Suppose a particular coin comes up heads twice as often as it comes up tails when we flip it a whole bunch of times.
   a. What is the probability of heads and what is the probability of tails of this coin.
   b. Write out a typical sequence of H's and T's corresponding to twenty flips of such a coin.

3. More "formally", the relative frequency method defines the probability of a coin coming up heads as the following limit

$$\text{Probability of Heads} = \lim_{n \to \infty} \frac{n_H}{n}$$

   where n_H equals the number of times the coin comes up heads and n equals the total number of times the coin is flipped.
   a. Could you ever be absolutely certain that your n is large enough. Explain
   b. Generalize on the definition above to explain why all probabilities p satisfy

$$0 \leq p \leq 1$$

4. Now let's suppose we flip a coin twice - an experiment with outcomes like HH and HT
   a. What is the **sample space** S of this experiment - the set of all possible outcomes when the coin is flipped twice. Note that we specify the outcomes in S as follows

$$S = \{HH, HT, \ldots\}$$

   b. What are the outcomes in the **event** A as follows

$$A = \{\text{All outcomes in S where the coin comes up heads exactly once}\}$$

   c. What are the outcomes in the event B as follows

B = {All outcomes in S where the coin comes up tails at least once}

5. From Problem (4) we know that there are four possible outcomes in the sample space S when we flip two coins. Now suppose that we can conclude from either symmetry arguments or from experimental data that all the outcomes in S are **equally likely**. Then given the events A and B as follows

   A = {All outcomes with exactly one head}

   B = {All outcomes with at least one tail}

   a. Find the probability of each outcome in S
   b. Find P(A) = Probability that the result of flipping the coins is an outcome in A
   c. Find P(B) = Probability that the result of flipping the coins is an outcome in B
   d. Find all the outcomes in the event C = A ∩ B = all outcomes that are in both A **and** B. Then find P(A ∩ B). Note that **A ∩ B** is referred to as the **intersection** of A and B
   e. Find all the outcomes in the event D = A   B = all outcomes that are in either A **or** B (or both). Then find P(A   B). Note that **A ∪ B** is referred to as the **union** of A and B.
   f. Now let's test the results of this problem with some experimental data. Flip a coin twice at least 50 times. Summarize your results in a Table
   g. Now make use of your experimental results in part (f) to check your results in (b) -(e). Use a Table like the following

Event	Calculated Prob	Measured Prob	% Difference

   Describe how well the calculated and experimental results agree

6. Two events E and F are said to be **mutually exclusive** if they both can't happen at the same time - if no outcome is in both events - if E ∩ F = φ where φ is the symbol for the empty set. **Memorize** this definition. Then find two mutually exclusive events for when a coin is flipped twice.

7. At this point the mathematicians like to jump in. Basically they tell us that no matter how we come up with values for our probabilities - whether from symmetry arguments, from experimental data or whatever - the probabilities need to satisfy the following properties: For all events A and B in a sample space S.

   (1) $0 \leq P(A) \leq 1$

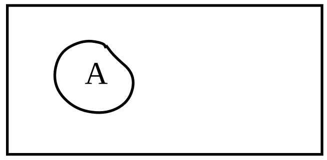
   (2) $P(S) = 1$

   (3) P(A   B) = P(A) + P(B)   if A ∩ B = φ

   It is from these properties that the mathematicians derive all the *official* properties of probability.

   a. Put into words what each of the above expressions says.
   b. Verify that your mutually exclusive events from Problem (6) satisfy condition (3).

8. The objective of this problem is to illustrate how Venn diagrams as follows



can be used to help us "visualize" probabilities. What we've done is simply draw a rectangle of **area one** and then draw a figure for event A with an area equal to the probability of A.

a. Draw a Venn diagram for event A with probability P(A) = 1/3
b. Draw a Venn diagram for events A and B with P(A) = 1/4 and P(B) = 1/3 if A and B are mutually exclusive
c. Repeat part (b) if A and B are not mutually exclusive

9. Let us now return to our coin flipping experiment of Problem (4) in which we flipped a coin twice to get equally likely outcomes like HH and HT
   a. Find the probability of the event D that after two flips of the coin there is at least one head. Compare with your experimental results from Problem (5).
   b. Find the probability of the event E that after two flips of the coin there is at least one tail. Compare with your experimental results from Problem (5).
   c. Now find the probability of the event F = D   E = the set of outcomes that are in event D **or** are in event E
   d. Now verify that your answer in part (c) is equal to

$$P(D \quad E) = P(D) + P(E) - P(D \cap E)$$

   Explain in words why this expression makes sense. Illustrate with a Venn diagram. **Memorize** this result.

10. Given the following expression

$$P(A) = P(A \cap B) + P(A \cap \overline{B})$$

   a. Explain in words why this result makes sense for all events A and B where $\overline{B}$ is the **complement** of B - all the outcomes in the sample space S that are not in B. Illustrate with a Venn diagram. **Memorize** this result
   b. See how well the experimental results in Problem (5) satisfy this relation. What can you conclude

11. Suppose we have two 1K resistors in series. What's the probability that the sum of the two resistors is 2K if each resistor has a probability of

p_1 = 1/3 of being exactly 1K
p_2 = 1/3 of being exactly 5% greater than 1K
p_3 = 1/3 of being exactly 5% less than 1K

Show all your work. Explain in words how you got your result. Assume that each combination of resistors is equally likely. Hint - start by making a Table for the nine possible outcomes in the sample space