ECE 307 - FOURIER TRANSFORMS - INVESTIGATION 8
FROM FOURIER SERIES TO FOURIER TRANSFORM - PART I

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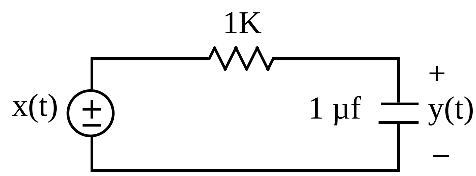
To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

So far so good. We now know how to find the steady state responses y(t) of linear circuits to periodic inputs x(t) like pulse trains by first expressing x(t) as a sum of complex exponentials and then making use of superposition to obtain y(t) as follows

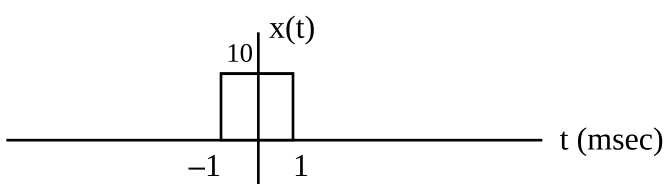
$$y(t) = \sum_{k=-\infty}^{\infty} X_k G(jk\omega_o) e^{jk\omega_o t}$$

The objective of this investigation is to make use of these frequency domain results to come up with a way to approximate the **zero state responses** of linear circuits to inputs that are **nonperiodic** signals like single pulses.

1. Let us begin with the following first order RC circuit

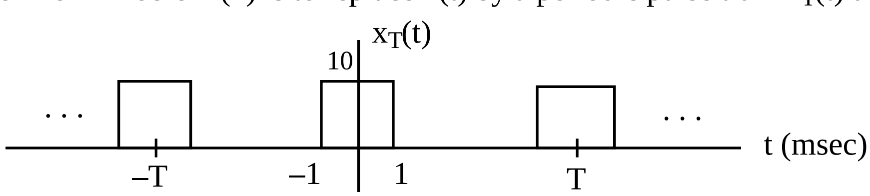


with **zero initial voltage** across the capacitor at time $t = -10^{-3}$ when the following single pulse arrives



   a. Find the circuit's time constant
   b. Make use of your result in part (a) to sketch y(t). Describe your graph and then explain why it looks the way it does

2. The trick that enables us to use frequency domain methods to find the response of a circuit to a single pulse like in Problem (1) is to replace x(t) by a periodic pulse train $x_T(t)$ as follows



that has a period T long enough for the circuit's response to for all practical purposes "decay to zero" between pulses

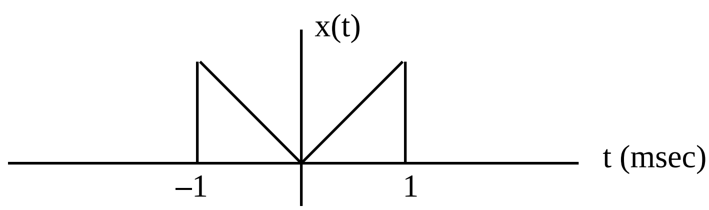
a. Explain in your own words how y(t) - the zero state pulse response to x(t) - can be approximated from $y_T(t)$ - the steady state response to the pulse train $x_T(t)$ when T is "large". Include sketches of x(t), y(t), $x_T(t)$ and $y_T(t)$
b. How large would you make T for this circuit. Explain how you got your result
c. Sketch $y_T(t)$ for your T in part (b)
d. Use SPICE to get a plot of the circuit's steady state response to your $x_T(t)$. Did the scheme work. How can you tell
e. Now make use of Mathcad to obtain a plot of the steady state $y_T(t)$ given by the following Fourier Series expansion

$$y_T(t) = \sum_{k=-\infty}^{\infty} X_k G(jk\omega_o) e^{jk\omega_o t}$$

over the time interval $-10^{-3} < t < T - 10^{-3}$ for your T in part (b). Be sure to include at least several lobes of spectral lines. Compare your result with your SPICE results. Explain any differences
f. What happens to the magnitudes of the y_K's as T is made larger

3. How large would you make T to get a good approximation of the pulse response of a circuit with the natural response $y_K(t) = K e^{-2000t} \cos(1000t + \theta)$ if the pulse width is a = 1 msec. Justify

4. Suppose we wanted to use this same frequency domain scheme to find the response of a circuit to another signal - say



How does changing the shape of x(t) like this affect the choice of the period T of the corresponding periodic signal