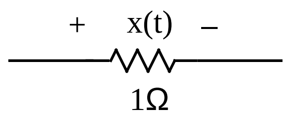
ECE 307 - COMPLEX FOURIER SERIES - INVESTIGATION 5
AVERAGE POWER OF PERIODIC SIGNALS

FALL 2000 **A.P. FELZER**

To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

The **average normalized power** of a periodic signal x(t) is by definition the average power when x(t) is the voltage across a 1Ω resistor as follows



The objective of this investigation is to find an equation for calculating the average normalized powers P_{av} of periodic signals x(t) in terms of their complex Fourier coefficients X_k.

1. We know from ECE 209 that the average power of a resistor R with periodic voltage x(t) is equal to the average of the power over one period as follows

$$Ave[p(t)] = Ave\left[\frac{x^2(t)}{R}\right] = \frac{Ave[x^2(t)]}{R} = \frac{\frac{1}{T}\int_T x^2(t)\,dt}{R}$$

   a. Make use of this result to show that the average normalized power of x(t) is given by

$$P_{av} = \frac{1}{T}\int_T x^2(t)\,dt$$

   b. Then explain in words why the average of the power over one period is equal to 1/T times the integral of $x^2(t)$

2. We also have from ECE 209 that the average normalized power of the periodic signal

$$x(t) = c_o + \sum_{k=1}^{\infty} c_k \cos(k\omega_o t + \theta_k) \qquad \text{is equal to} \qquad P_{av} = c_o^2 + \frac{1}{2}\sum_{k=1}^{\infty} c_k^2$$

   a. Show that $c_o^2 = |X_o|^2$

   b. Show that $\frac{1}{2}c_1^2 = |X_{-1}|^2 + |X_1|^2$

   c. Show that $\frac{1}{2}c_k^2 = |X_{-k}|^2 + |X_k|^2$

   d. Now make use of your results to show that the average normalized power of the periodic signal

$$x(t) = c_o + \sum_{k=1}^{\infty} c_k \cos(k\omega_o t + \theta_k) = \sum_{k=-\infty}^{\infty} X_k e^{jk\omega_o t}$$

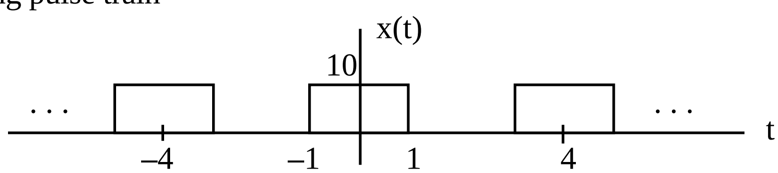
is equal to $P_{av} = \sum_{k=-\infty}^{\infty} |X_k|^2$

3. Find the average normalized powers P_{av} of
   a. $x(t) = 4e^{j\pi/3}e^{-j100t} + 4e^{-j\pi/3}e^{j100t}$
   b. $x(t) = 4e^{j\pi/3}e^{-j100t} + 3 + 4e^{-j\pi/3}e^{j100t}$
   c. $x(t) = 2e^{-j}e^{-j200t} + 4e^{j\pi/3}e^{-j100t} + 3 + 4e^{-j\pi/3}e^{j100t} + 2e^{j}e^{j200t}$

4. Combining the results Problems (1) and (2) we have that

$$P_{av} = \frac{1}{T}\int_T x^2(t)\,dt = \sum_{k=-\infty}^{\infty} |X_k|^2$$

We refer to this result as **Parseval's Theorem**. **Memorize** this result. It shows how the average normalized power can be calculated as an integral in the time domain as well as a sum in the frequency domain. The objective of this problem is to see how many harmonics must be included to get a good approximation of the average normalized power for a typical pulse train
   a. Make use of the integral expression above to find the average normalized power P_{av} of the following pulse train

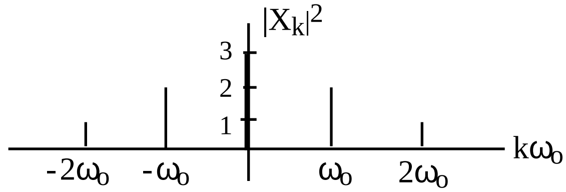


   b. Now keep adding the powers of higher harmonics until the sum of their powers is at least 97% of the total average normalized power. A program like Mathcad can be particularly helpful here. How many harmonics (including ones equal to zero) did you need
   c. How wide must be the passband of an ideal lowpass filter to pass 97% of the power of this x(t)

5. What would you expect double-sided power spectral plots are. Draw the double-sided normalized power spectral plot of

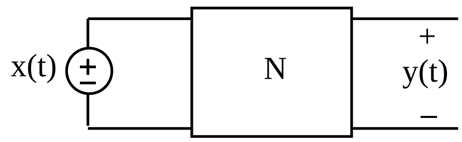
$$x(t) = 2e^{-j}e^{-j200t} + 4e^{j\pi/3}e^{-j100t} + 3 + 4e^{-j\pi/3}e^{j100t} + 2e^{j}e^{j200t}$$

6. Find the average normalized power P_{av} of the periodic signal x(t) with double-sided power spectral plot

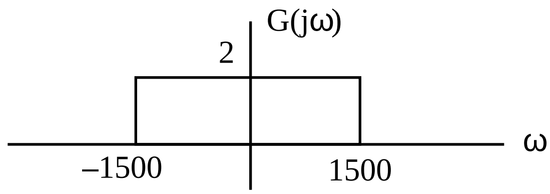


7. Now suppose the periodic signal x(t) = 2 + 3 cos (1000t) + cos (2000t) is the input to the

following circuit



with transfer function



Note that the phase of this idealized circuit is everywhere equal to zero

a. Draw a double-sided spectral plot and a double-sided normalized power spectral plot of x(t)
b. Find the average normalized power of x(t)
c. Find the steady state y(t)
d. Draw a double-sided spectral plot and a double-sided power spectral plot of y(t)
e. Find the average normalized power of y(t)

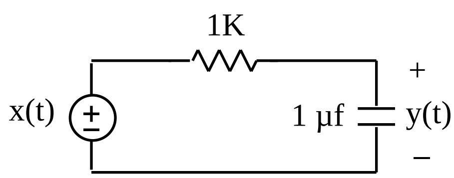
8. Given that $x(t) = 2 + 3\cos(1000t) + \cos(2000t)$ is the input to a linear circuit with transfer function

$$G(j\omega) = \frac{j\omega}{j\omega + 1000}$$

and output y(t)

a. Find the steady state y(t)
b. Draw a double-sided spectral plot and a double-sided power spectral plot of y(t)
c. Find the average normalized power of y(t)

9. Given that $x(t) = 2 + 3\cos(1000t) + \cos(2000t)$ is the input to the following circuit



a. Find the steady state y(t)
b. Draw a double-sided spectral plot and a double-sided power spectral plot of y(t)
c. Find the average normalized power of y(t)

10. Show that if the periodic signal $x(t) = \sum_{k=-\infty}^{\infty} X_k e^{jk\omega_o t}$ is the input of a circuit with transfer function G(jω) then the average normalized power of the steady state response y(t) is

$$P_{av} = \sum_{k=-\infty}^{\infty} |Y_k|^2 = \sum_{k=-\infty}^{\infty} |G(jk\omega_o)|^2 |X_k|^2$$

This is an important result. **Memorize** it