ECE 307 - COMPLEX FOURIER SERIES - INVESTIGATION 4
COMPLEX FOURIER SERIES IN STEADY STATE ANALYSIS

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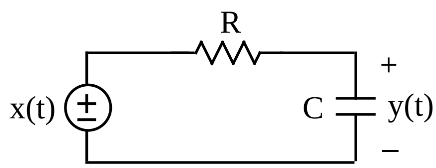
To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

We know from ECE 209 that if a linear circuit like the following

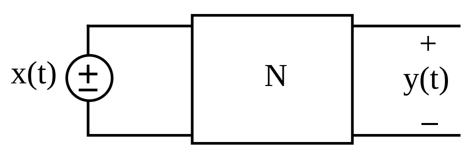


is in the sinusoidal steady state then the phasors are related as follows

$$Y(j\omega) = G(j\omega)\, X(j\omega)$$

We also know that superposition holds - that the steady state response of a linear circuit to a sum of sinusoids is the sum of the circuit's sinusoidal steady state respsonses to the individual sinusoids. The objective of this investigation is to express this result in terms of complex exponentials.

1. The objective of this first problem is to find general equations for the complex Fourier coefficients Y_{-1} and Y_1 of the steady state response of the following circuit



in terms X_{-1} and X_1 of the first harmonic of x(t)

a. Make use of your results from Problem (7) of Investigation 1 and the fact that the phasors $Y(j\omega_o)$ and $X(j\omega_o)$ of the first harmonics of x(t) and y(t) are related by $Y(j\omega_o) = G(j\omega_o)\, X(j\omega_o)$ to find expressions for the complex Fourier coefficient Y_1 of the steady state y(t) in terms of the circuit's transfer function $G(j\omega)$ and the complex Fourier coefficient X_1 of the first harmonic of the input.

b. Now make use of the fact that $Y_{-1} = Y_1$, the fact that the complex conjugate of a product is equal to the product of the complex conjugates with $(z_1 z_2)^* = z_1^* z_2^*$ and also the fact that transfer functions $G(j\omega)$ can be shown to satisfy $G^*(j\omega) = G(-j\omega)$ to show that

$$Y_{-1} = G(-j\omega_o) X_{-1}$$

c. Find Y_{-1} and Y_1 if the first harmonic of x(t) is 5 cos (1000t) and $G(j\omega) = \dfrac{1000}{j\omega + 1000}$

2. Generalize on your results in Problem (1) to find expressions for the complex Fourier coefficients Y_k and Y_{-k}

3. Make use of your results in Problems (1) and (2) to show that if

$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{jk\omega_o t}$$

is the periodic input to a linear circuit with transfer function G(jω), then its steady state response will be

$$y(t) = \sum_{k=-\infty}^{\infty} Y_k e^{jk\omega_o t} = \sum_{k=-\infty}^{\infty} G(jk\omega_o) X_k e^{jk\omega_o t}$$

Memorize this result.

4. Write out the terms in the sum

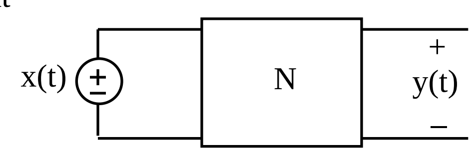
$$y(t) = \sum_{k=-2}^{2} G(jk\omega_o) X_k e^{jk\omega_o t}$$

5. Find the steady state response of a circuit N with input

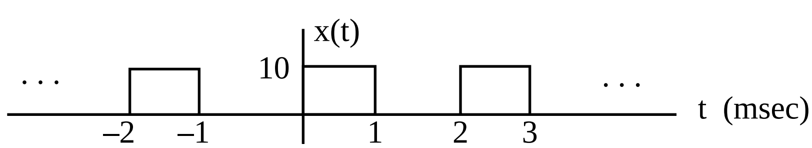
$$x(t) = 3\,e^{-j\pi/6}\ e^{-j200t} + 4\,e^{j\pi/3}\ e^{-j100t} + 2 + 4\,e^{-j\pi/3}\ e^{j100t} + 3\,e^{j\pi/6}\ e^{j200t}$$

and transfer function $G(j\omega) = \dfrac{100}{j\omega + 100}$ as a sum of complex exponentials. Draw the double–sided spectral plots of the input and output.

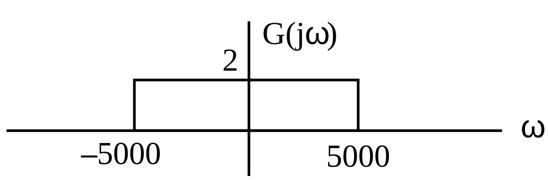
6. Given the following circuit



Find and draw the double-sided spectral plot of the complex Fourier Coefficients Y_k of the steady state response if x(t) is the pulse train

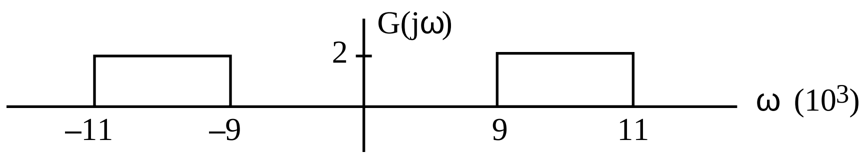


and N has the lowpass frequency response



Note that we must now draw G(jω) for negative as well as positive frequencies. Note also the phase of this idealized circuit is everywhere equal to zero. Write out the corresponding expression for y(t). Hint - make use of the expression for the X_k's derived for pulse trains in the last investigation.

7. Repeat Problem (6) if the circuit N has the bandpass frequency response



8. Find the complex Fourier coefficients Y_k of the first three harmonics of the steady state output of the following circuit with x(t) the pulse train of Problem (6)