# ECE 307 - COMPLEX FOURIER SERIES - INVESTIGATION 3
# COMPLEX FOURIER COEFFICIENTS OF PULSE TRAINS

**FALL 2000** **A.P. FELZER**

To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

In the previous investigation we came up with the following expression

$$X_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_o t}\,dt$$

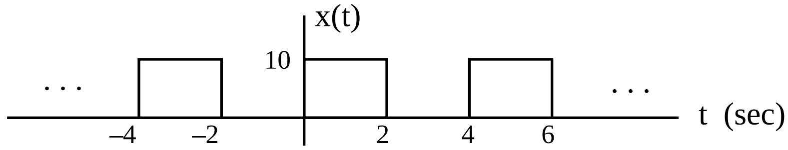
for calculating the Complex Fourier Series Coefficients $X_k$ of periodic signals x(t). **Memorize** this expression. The objective of this investigation is to calculate these coefficients for pulse trains – not only because the calculations are fairly straightforward but also because pulse trains are so common in electrical systems – especially digital systems. We will then make use of our pulse train results to surmise what's going on in more general situations.

1. Given the following periodic signal

$$x(t) = 2e^{-j\pi/4}e^{-j200t} + 3e^{j\pi/3}e^{-j100t} + 4 + 3e^{-j\pi/3}e^{j100t} + 2e^{j\pi/4}e^{j200t}$$

   a. Find $\omega_o$, $f_o$ and T
   b. Find $X_{-2}$, $X_{-1}$, $X_0$, $X_1$ and $X_2$

2. Given the following pulse train



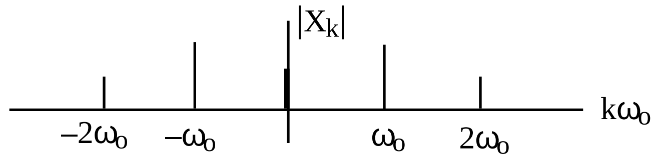

   a. Find $X_o$
   b. Verify that the complex Fourier Coefficients $X_k$ for $n \neq 0$ are given by

$$X_k = 5\frac{\sin(k\pi/2)}{k\pi/2}\,e^{-jk\pi/2}$$

   Hint – make use of the fact that $e^{-jk\pi} - 1 = e^{-jk\pi/2}\left(e^{-jk\pi/2} - e^{jk\pi/2}\right)$

   c. Now calculate $X_{-5}, \cdots, X_{-1}, X_1, \cdots, X_5$

3. As is often the case, it is nice to have graphs that show us how things vary. So for the complex coefficients you found in Problem (1)
   a. Make a discrete spectral plot like the following



of the magnitudes $|X_k|$ versus $k\omega_o$ for $k = -2$ to $n = 2$

b. Make a discrete spectral plot of the phases $X_k$ versus $k\omega_o$ for $k = -2$ to $k = 2$. **Plot the phases** in the range $-\pi$ to $\pi$. Note that these plots are referred to as **double sided spectral plots** because they are plotted for both negative and positive frequencies.

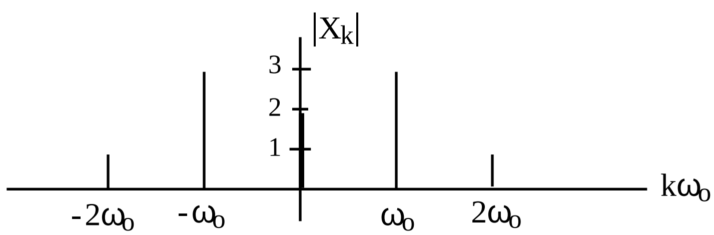
4. Given the pulse train of Problem (2) with an expression for the $X_k$'s as given in part (b)

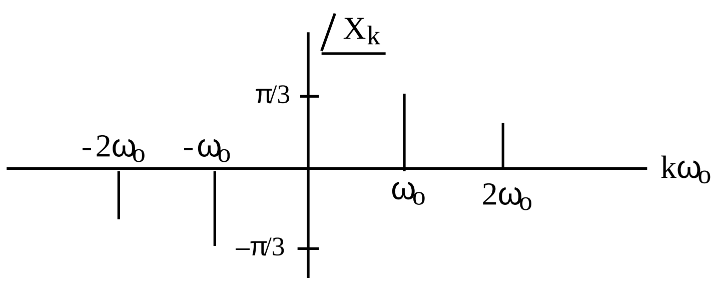
   a. Use a program like Mathcad or Matlab to obtain graphs of

$$\sum_{k=-m}^{m} X_k e^{jk\omega_o t} \qquad \text{for } m = 1, 3, 5, 7 \text{ and } 9$$

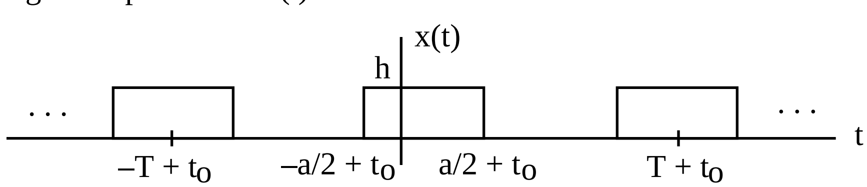
   b. Describe and then explain what's going on as k increases

5. Now let's go in the opposite direction. Obtain an expression for the signal x(t) with the following spectral plot where $\omega_o = 200$




6. Verify that a general pulse train x(t) as follows




that has a magnitude h, pulse width a, period T, fundamental frequency $f_o = 1/T$ and is delayed by $t_o$ seconds has complex Fourier coefficients given by

$$X_k = \frac{ha}{T} \frac{\sin(k\pi f_o a)}{(k\pi f_o a)} e^{-j2\pi k f_o t_o}$$

Then verify that this equation gives the same result you got for $X_k$ in Problem (2b).