ECE 307 - CONVOLUTION - INVESTIGATION 26
FREQUENCY DOMAIN CONVOLUTION

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To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

The objective of this investigation is to tie the knot linking convolution and frequency domain analysis

1. We know from our investigations on convolution that the zero state response y(t) of a circuit with input x(t) and impulse response h(t) is given by y(t) = x(t)∗h(t). And we know from our

   investigations on Fourier Transforms that $Y(\ ) = H(\ )X(\ )$ where H( ) – the circuit's frequency response – is the Fourier Transform of the circuit's impulse response. Make use of these results to show that

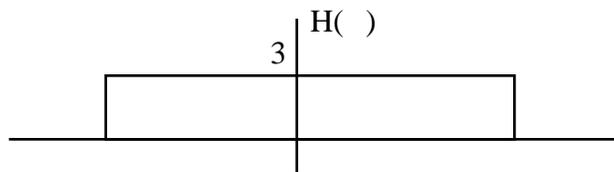$$F[x(t) \ast h(t)] = H(\ )X(\ )$$

   Memorize this relation.

2. Sketch $Y(\ ) = F[x(t) \ast h(t)]$ for a circuit with input x(t) that has the Fourier Transform
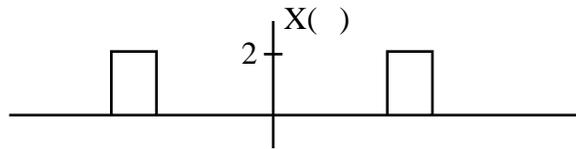
   and transfer function

   Explain how you got your result

3. Sketch $Y(\ ) = F[x(t) \ast h(t)]$ for a circuit with periodic input $x(t) = \cos(1000t)$ and impulse response $h(t) = 2000 \exp(-1000t)u(t)$. Then sketch the corresponding y(t). Why is this y(t) equal to the steady state rather than "zero state response" of this circuit. What happens to the transient response

4. Surprising as it may seem at first, the symmetry of the general convolution integral as follows
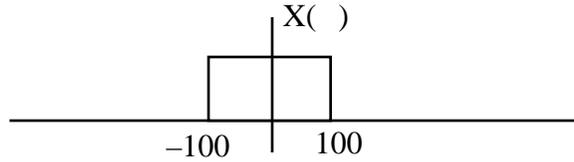$$F[x(t) \ast y(t)] = X(\ )Y(\ )$$
   implies the dual result

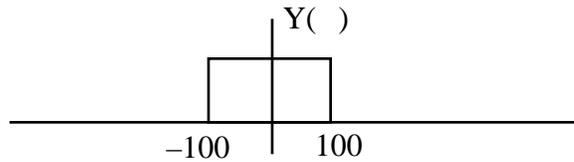$$F[x(t)y(t)] = \frac{1}{2} X(\omega) \quad Y(\omega) = \frac{1}{2} \int X(\ )Y(\omega - \ )d$$

with the convolution integral having limits of integration from $\ = -\ $ to $\ = \ $. Write out in words what each of these expressions says. **Memorize** this relation.

5. Sketch F[x(t) y(t)] in each of the following cases if X( ) is given by

X( )
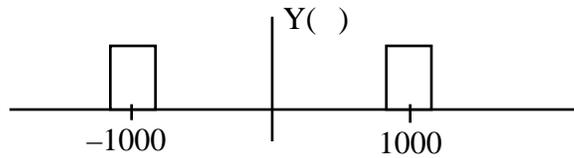
−100   100

and

   a. Y( ) is given by

Y( )

−100   100

   b. Y( ) is given by

Y( )

−1000   1000

   c. y(t) = cos 1000t