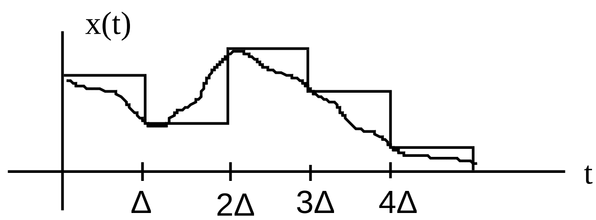
ECE 307 - CONVOLUTION - INVESTIGATION 25
CONVOLUTION INTEGRAL

FALL 2000 **A.P. FELZER**

To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

The objective of this investigation to extend the results of the two previous investigations to signals that are not sums of pulses - but can be approximated by such sums as follows



Now, as we saw in the last investigation, the zero state response to a sum of pulses

$$x_{\Delta}(t) = \sum_{n=0}^{N-1} x_{\Delta}(n\Delta)\Delta p_{\Delta}(t - n\Delta)$$

is the following sum of zero state pulse responses

$$y_{\Delta}(t) = \sum_{n=0}^{N-1} x_{\Delta}(n\Delta) h_{\Delta}(t - n\Delta)\,\Delta$$

Now, by inspection it seems clear that the approximations $x_{\Delta}(t) \approx x(t)$ and $y_{\Delta}(t) \approx y(t)$ should get better and better as Δ gets smaller and smaller. In fact, it can be shown that as $\Delta \rightarrow 0$ the summation becomes the following integral for y(t)

$$y(t) = \int_{-\infty}^{t} x(\tau) h(t - \tau)\, d\tau$$

as $h_{\Delta}(t)$ metamorphizes into the circuit's impulse response h(t).

Integrals like that above for y(t) are referred to as **convolution integrals**. Formally we define the convolution of x(t) and w(t) to be the following integral

$$y(t) = \int_{-\infty}^{\infty} x(\tau) w(t - \tau)\, d\tau = x(t)\ \ w(t)$$

with limits of integration from $\tau = -\infty$ to $\tau = \infty$. Note that the asterisk $*$ means convolution and not multiply. This general expression for the convolution integral reduces to

$$y(t) = \int_{-\infty}^{t} x(\tau) h(t - \tau)\, d\tau = x(t)\ \ h(t)$$

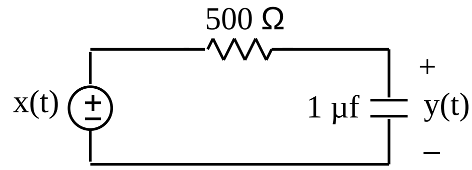
when w(t) = h(t) since the impulse responses of real circuits with zero initial conditions satisfy

$$h(t) = 0 \quad \text{for } t < 0$$

This is so since real circuits cannot have outputs until t = 0 when the impulse occurs. **Memorize** the above general integral for the convolution of x(t) and w(t)

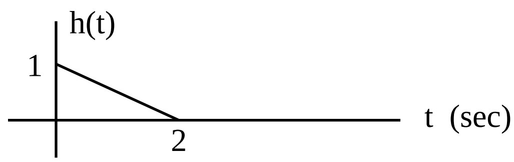
1. Evaluate the convolution integral to find the step response s(t) of a circuit with impulse response $h(t) = 200e^{-1000t}\,u(t)$

2. Given the following circuit



   a. First find the circuit's impulse response h(t)
   b. Then use convolution to find an expression for the circuit's step response s(t)

3. The objective of this problem is to see the *geometric meaning* of the convolution integral. Let's suppose we want to use convolution to find the step response s(t) of a circuit with impulse response h(t) as follows at the time t = 3 seconds



   a. First sketch u(τ), h(τ), h(–τ) and h(t – τ) versus τ for t = 3
   b. Then draw a graph of u(τ)h(3 – τ)
   c. And finally obtain

$$s(3) = \int_{-\infty}^{3} u(\tau)h(3-\tau)\,d\tau$$

   by calculating the area under the curve you plotted in part (b)
   d. Describe in words the steps involved in calculating s(3). What, in particular, is flipped, what is shifted, what is multiplied and what is integrated
   e. Describe how this scheme can be used to calculate s(t) at other times t
   f. Make use of your result in part (e) to sketch a graph of s(t).

4. Now suppose we want to find the step response of a circuit with impulse response $h(t) = 200e^{-1000t}\,u(t)$ at time t = 2 msec
   a. First sketch u(τ), h(τ), h(–τ) and h(t – τ) versus τ for t = 2 msec
   b. Then sketch $u(\tau)\,h(2x10^{-3} - \tau)$
   c. And finally get an approximation for

$$s(2x10^{-3}) = \int_{-\infty}^{2x10^{-3}} u(\tau)\,h(2x10^{-3}-\tau)\,d\tau$$

   by estimating the area under the curve you sketched in part (b)
   d. Explain how you would graphically find s(t) at other times t
   e. Make use of your result in part (d) to sketch s(t). Explain how you got your result
   f. Describe what's happening to the step response as t increases