ECE 307 - CONVOLUTION - INVESTIGATION 24
ZERO STATE PULSE RESPONSES - PART II

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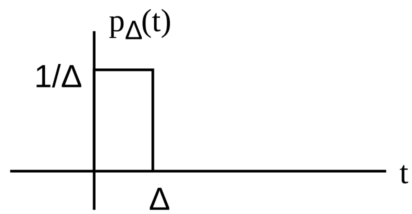
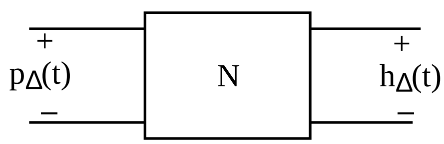
To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

The objective of this investigation is to come up with a general expression for the zero state response of a circuit in the case when its input is equal to a sum of pulses. We shall use the notation $p_\Delta(t)$ for a pulse of area one with width Δ and magnitude $1/\Delta$ as follows

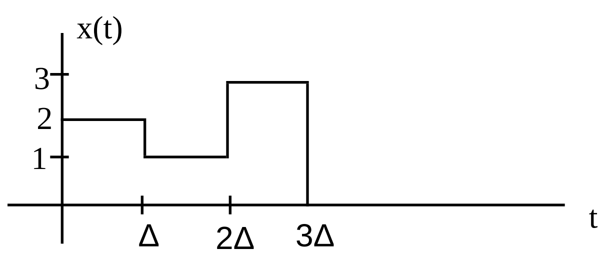


and $h_\Delta(t)$ for the corresponding zero state pulse response as follows



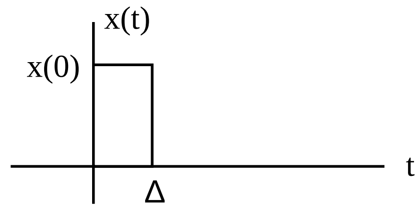
1. To begin sketch each of the following pulses for $\Delta = 1/4$
   a. $p_\Delta(t)$
   b. $p_\Delta(t - \Delta)$
   c. $p_\Delta(t - 2\Delta)$
   d. $\Delta p_\Delta(t - 2\Delta)$
   e. $3\Delta p_\Delta(t - 2\Delta)$
   f. $2\Delta p_\Delta(t) + 3\Delta p_\Delta(t - \Delta)$

2. Express the following signal x(t)



as a sum of pulses of the form $x(n\Delta)\Delta p_\Delta(t - n\Delta)$

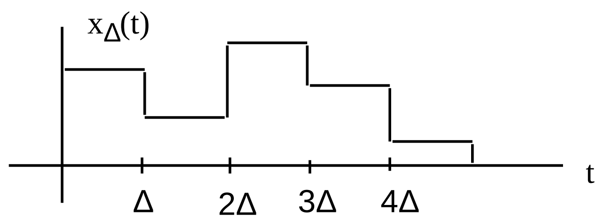
3. Show that the zero state response of a circuit to the following input



is $y(t) = x(0)\Delta h_\Delta(t)$. Hint - express the input as $x(t) = x(0)\Delta p_\Delta(t)$

4. Express a circuit's zero state response to each of the following inputs in terms of $h_\Delta(t)$
   a. $p_\Delta(t)$
   b. $p_\Delta(t - \Delta)$
   c. $p_\Delta(t - 2\Delta)$
   d. $2p_\Delta(t - 2\Delta)$
   e. $p_\Delta(t - \Delta) + 2p_\Delta(t - 2\Delta)$

5. Suppose that the input to a given circuit is a sum of pulses as follows



with pulse magnitudes $x(0)$, $x(\Delta)$, $x(2\Delta)$, $x(3\Delta)$, $x(4\Delta)$

   a. Write out an expression for $x_\Delta(t)$ as a sum of pulses $x(n\Delta)\Delta p_\Delta(t - n\Delta)$
   b. Make use of your result in part (a) to express

$$y_\Delta(t) = \text{Zero state response of the circuit to } x(t)$$

   as a sum of terms of the form $x(n\Delta)\Delta h_\Delta(t - n\Delta)$.

   c. Show that your expression in part (b) can be expressed as the following sum

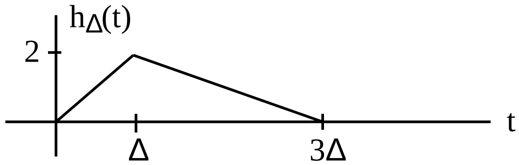
$$y_\Delta(t) = \sum_{n=0}^{4} x(n\Delta)h_\Delta(t - n\Delta)\Delta$$

   which we refer to as a **convolution sum**. It tells us that the zero state response at time t is - by superposition - equal to the sum of the responses to the pulses that add up to $x_\Delta(t)$

   d. Make use of your result in part (c) to obtain an expression for $y_\Delta(5\Delta)$
   e. Make use of your result in part (d) to calculate $y_\Delta(5\Delta)$ for $\Delta = 0.5$ and the following values of $x(n\Delta)$ and $\Delta h_\Delta(n\Delta)$. Put the results of your calculations in a Table

$x(0) = 2$, $x(\Delta) = 1$, $x(2\Delta) = 3$, $x(3\Delta) = 2$, $x(4\Delta) = 1$, $x(5\Delta) = 0$
$h_\Delta(\Delta) = 4$, $h_\Delta(2\Delta) = 2$, $h_\Delta(3\Delta) = 1$, $h_\Delta(4\Delta) = 0.5$, $h_\Delta(5\Delta) = 0.2$

6. Make use of the convolution sum to find the step response at $t = 5\Delta$ of a circuit with a pulse response as follows



when $\Delta = 0.5$. Find $h_\Delta(0)$, $h_\Delta(\Delta)$, $h_\Delta(2\Delta)$, $h_\Delta(3\Delta)$ and $h_\Delta(4\Delta)$ and then draw and make use of graphs like those in Problem (6).

7. Now suppose we have an input x(t) that is a general sum of pulses as follows

$$x(t) = x(0)\Delta p_\Delta(t) + x(\Delta)\Delta p_\Delta(t - \Delta) + x(2\Delta)\Delta p_\Delta(t - 2\Delta) + \cdot \cdot \cdot + x(n\Delta)\Delta p_\Delta(t - n\Delta)$$

   a. Express the zero state response $y_\Delta(t)$ as a sum with a Σ
   b. What do you think happens to $p_\Delta(t)$, $h_\Delta(t)$ and the summation for $y_\Delta(t)$ as $\Delta \to 0$

8. Now sketch each of the following as functions of τ. Note that you may find it helpful in parts (c)–(f) to sketch each factor separately and then multiply them together.
   a. $x_1(\tau) = u(\tau - 3)$
   b. $x_2(\tau) = u(3 - \tau)$
   c. $x_3(\tau) = 2 \exp(-\tau)\, u(\tau)$
   d. $x_4(\tau) = 2 \exp(\tau)\, u(-\tau)$
   e. $x_5(\tau) = 2 \exp(-1 + \tau)\, u(1 - \tau)$
   f. $x_6(\tau) = 2 \exp(-1 + \tau)\, u(1 - \tau)\, u(\tau)$