ECE 307 - CONVOLUTION - INVESTIGATION 23
ZERO STATE PULSE RESPONSES - PART I

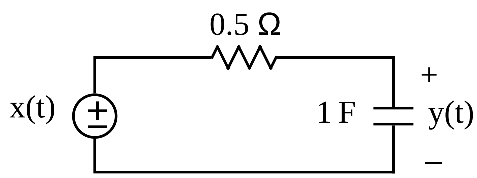
FALL 2000 **A.P. FELZER**

To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

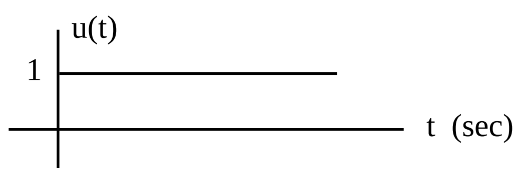
From our previous investigations we've learned how to use frequency domain analysis to find steady state responses of linear circuits to periodic inputs. We express the inputs as sums of sinusoids and then find the steady state responses by superposition - by adding up the sinusoidal steady state responses of the individual sinusoids. With Fourier Transforms we do basically the same thing with nonperiodic inputs.

The objective of this and the next two investigations is to show how a similar time domain method called *convolution* can be used to find zero state responses - the responses of circuits with zero initial conditions. The main difference between frequency domain analysis and time domain analaysis with convolution is that in convolution we express the inputs as sums of pulses rather than sinusoids.

1. Beginning with the following circuit



   a. First find and sketch its **step response s(t)** - its zero state response to the unit step u(t) given by



   Explain what's happening in the circuit

   b. What would you expect is the zero state response to $x(t) = 2u(t)$. Make a sketch. Carry out the analysis to check your result. How about when $x(t) = Ku(t)$

   c. Now sketch the zero state unit **pulse response $h_1(t)$** - its zero state response to the unit pulse $p_1(t)$ given by

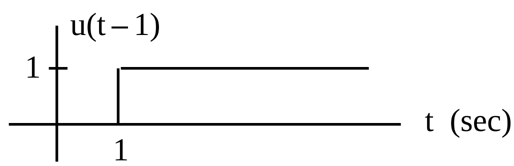
   Describe what's going on in the circuit during the time $0 \leq t \leq 1$ when the pulse is ON and then for $t > 1$ when the pulse is OFF. Be sure to indicate $h_1(1)$ on your graph and to draw and label the corresponding circuits when the pulse is ON and when it's OFF.

Then show that

$$h_1(t) = \begin{cases} 1 - e^{-2t} & 0 \le t \le 1 \\ h_1(1)e^{-2(t-1)} & t > 1 \end{cases}$$

d. Use Mathcad to get a plot of the circuit's zero state response to $p_1(t)$
e. What would you expect is the zero state response to $x(t) = 2p_1(t)$ in terms of $h_1(t)$. Make a sketch. Justify your result.
f. Generalize on your result in part (e) to find the zero state resonse to $x(t) = Kp_1(t)$ in terms of $h_1(t)$

2. Let us now look at steps and pulses that are delayed - that start at times later than $t = 0$
   a. Justify the fact that if s(t) is a circuit's step response, then its response to the delayed step u(t - 1) as follows



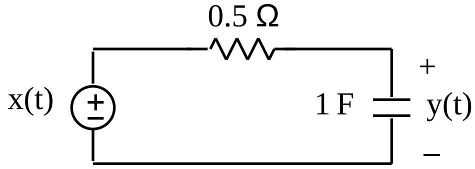
   is s(t - 1). Sketch s(t - 1)
   b. Express the circuit's zero state response to $p_1(t - 1)$ in terms of $h_1(t)$. Make sketches of your results
   c. What would you expect is the zero state response to the general delayed pulse $p_1(t - t_o)$ delayed by t_o

3. The objective of this problem is to demonstrate that **zero state responses of linear circuits satisfy superposition**
   a. Show that if $y_1(t)$ is the zero state response to the input $x_1(t)$ of a circuit with the following differential equation

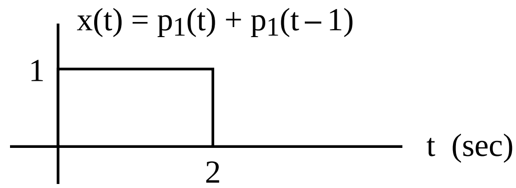
$$y' + 1000y = 100x$$

   and $y_2(t)$ is the zero state response when the input is $x_2(t)$ then the zero state response of the circuit to the input $x(t) = x_1(t) + x_2(t)$ is $y(t) = y_1(t) + y_2(t)$. Do this by showing that
      (i) y(t) satisfies the zero initial condition $y(0-) = 0$
      (ii) When y(t) is substituted into $y' + 100y$ it gives $100x(t)$
   b. Generalize on your result from part (a) to find the zero state response of a linear circuit with input $x(t) = x_1(t) + x_2(t) + \cdots + x_n(t)$. **Memorize** this result

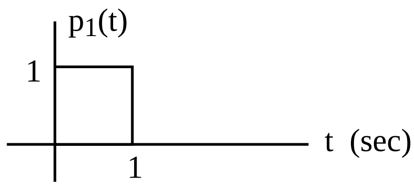
4. Let us now apply the results of Problem (3) to find the zero state response of our circuit



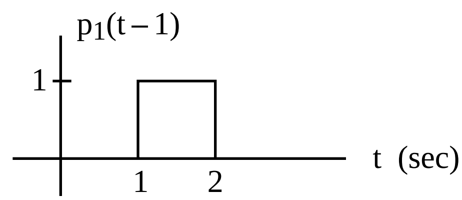
to the input $x(t) = p_1(t) + p_1(t - 1)$ as follows



where $p_1(t)$ and $p_1(t - 1)$ are given by

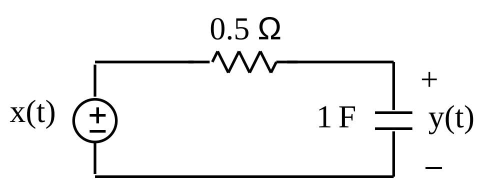


and

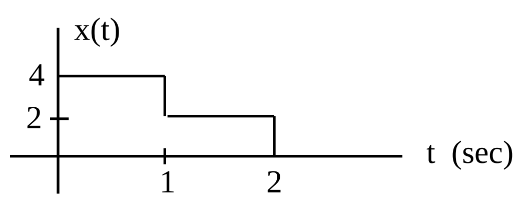


a. Sketch the zero state pulse responses $h_1(t)$ and $h_1(t - 1)$ to $p_1(t)$ and $p_1(t - 1)$
b. How would you expect the sum of these zero state responses to be related to the zero state response of the circuit to the sum $x(t) = p_1(t) + p_1(t - 1)$
c. Make use of the results in Problem (1c) to analytically verify that the sum of the zero state responses $h_1(t)$ and $h_1(t - 1)$ at time $t = 3$ sec is in fact equal to the circuit's zero state response to x(t) at time $t = 3$ sec
d. Make use of Mathcad to verify that $h_1(t) + h_1(t - 1)$ does in fact look like the circuit's response to x(t)

5. Given the following circuit



with input as follows



a. First write out x(t) in terms of unit pulses $p_1(t)$
b. Then write out the zero state response of y(t) in terms of unit pulse responses $h_1(t)$
c. Then make use of your results in part (b) to sketch the zero state response of y(t)
d. And then make use of Mathcad to obtain a graph of y(t)