ECE 307 - UNILATERAL LAPLACE TRANSFORM - INV 22
CIRCUITS WITH NONZERO INITIAL CONDITIONS

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To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

Up to now all of our inputs

$$x(t) = f(t)\,u(t)$$

have been zero up to time $t = 0$ and so all of our circuits have had zero initial conditions. The objective of this investigation is to introduce the **unilateral LaPlace Transform** for analyzing circuits and systems with nonzero initial conditions.

1. The **unilateral** LaPlace Transform as **defined** by

$$\int_{0-}^{\infty} e^{-st} f(t)\,dt$$

is basically the same as its bilateral cousin except that its integral starts at time $t = 0-$ instead of at time $t = -\infty$. As a practical matter, we don't have to do any recalculating to obtain the unilateral transforms of signals multiplied by u(t) like

$$u(t),\ \cos(\omega t)\,u(t),\ \ldots$$

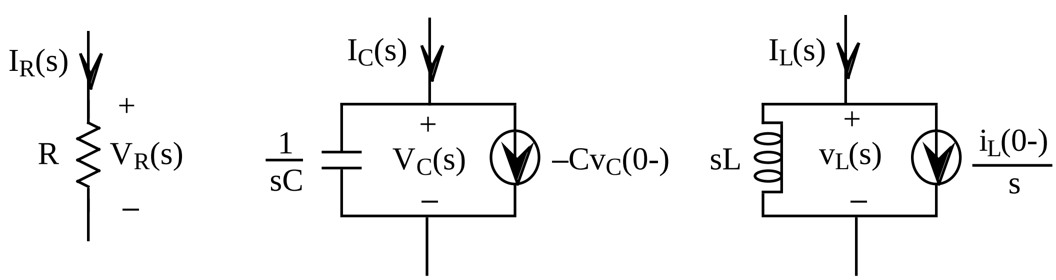
since for these functions the unilateral and bilateral LaPlace Transforms are equal. The only transform that we must recalculate is that of the derivative which by straightforward integration by parts can be shown to equal

$$\mathbf{L}\left[\frac{dx(t)}{dt}\right] = sX(s) - x(0-)$$

where $x(0-)$ = the value of x(t) just before time $t = 0$. **Memorize** this result. It's what people usually mean when they say they've taken the LaPlace Transform of a derivative.

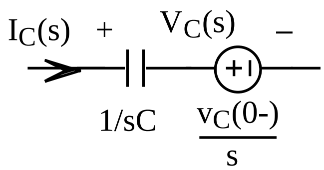
   a. Use the above result to find the relations between the unilateral LaPlace Transforms of the voltages and the currents of resistors, capacitors and inductors. How are these relations different from those for the bilateral LaPlace Transform

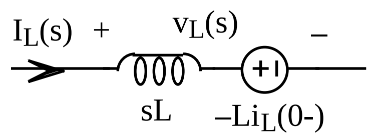
   b. Verify that the following are unilateral transformed circuits



   for resistors, capacitors and inductors by finding their I(s)-V(s) relations and then verifying that they agree with your results in part (a)

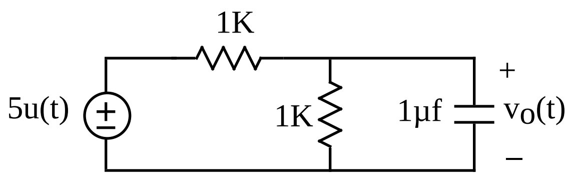
   c. Verify that the following are also unilateral transformed circuits





for capacitors and inductors

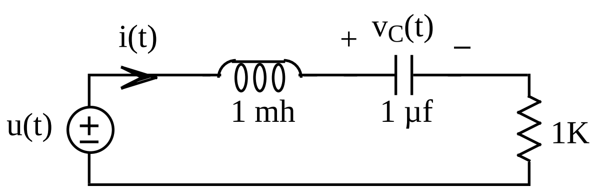
2. Make use of the unilateral LaPlace Transform to find and then sketch the response of the following circuit



$v_O(0–) = –2$ volts

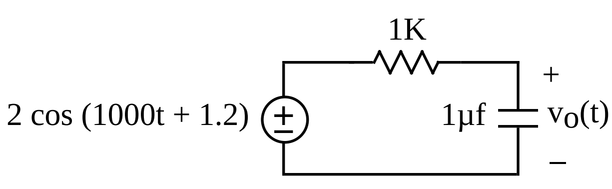
3. Make up and then analyze a circuit with a controlled source or op amp that has nonzero initial conditions

4. Find I(s) in the following circuit



$i(0–) = –5$ ma $\quad v_C(0–) = 2$ volts

5. Find the sinusoidal steady state response of the following circuit



$v_O(0–) =5$ volts

What affect did the initial condition have on your result

6. Show that the unilateral LaPlace Transform of the second derivative of v(t) is given by

$$L\left[\frac{d^2v(t)}{dt^2}\right] = s^2V(s) - sv(0-) - v'(0-)$$

Hint – make use of the fact that $L\left[\frac{d^2v(t)}{dt^2}\right] = L\left[\frac{d}{dt}\left(\frac{dv(t)}{dt}\right)\right]$

7. Find V(s) for

$$v'' + 1000v' + 10^6v = 10^5u(t) \qquad v'(0–) = 100,\ v(0–) = 2$$

8. Find $G(s) = V_o(s)/V_{in}(s)$ for