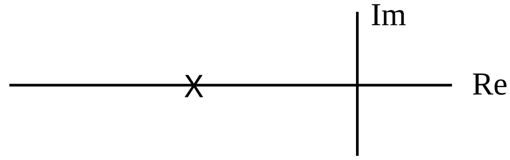
ECE 307 - POLES AND ZEROS - INVESTIGATION 21
POLES AND ZEROS AND FREQUENCY RESPONSES - PART II

FALL 2000 **A.P. FELZER**

To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

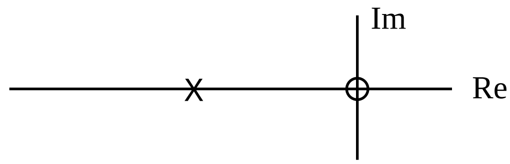
As we've seen in the last several investigations, poles and zeros are very useful because of the direct connection between them and natural and frequency responses. The objective of this investigation is to see how a circuit's frequency response changes as its poles and zeros move around in the complex plane

1. Given a circuit with the following pole-zero diagram



   a. Draw a series of sketches to illustrate what happens to the magnitude of the frequency response as the magnitude of the pole p is increased. Assume that in each case the circuit is such that $|G(j0)| = 1$ no matter what the value of p.
   b. What's an example of a circuit that has a transfer function with this pole-zero diagram

2. Repeat Problem (1) for



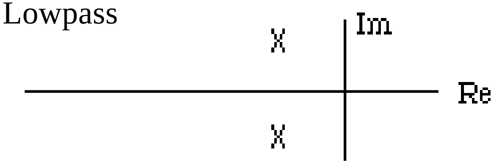
3. Let us now consider a second order circuit with transfer function G(s) as follows

$$G(s) = \frac{K}{(s + a - j1000)(s + a + j1000)}$$

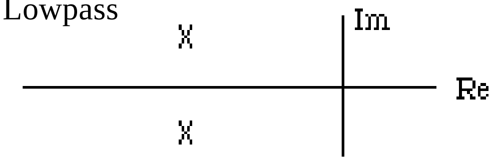
   a. First draw the pole-zero diagrams and sketch what you would expect $|G(j\omega)|$ looks like for values of a satisfying $a < 1000$, $a = 1000$ and $a > 1000$
   b. Now express $|G(j\omega)|$ as a function of $DP1(\omega)$ and $DP2(\omega)$ – the distances in the complex plane between the poles p_1 and p_2 and the complex number $j\omega$ with imaginary part ω. And then use a plotting calculator or computer to obtain plots of $|G(j\omega)|$ for the different values of the parameter *a*. Adjust K in each case so that $|G(j0)| = 1$
   c. Were your conjectures in part (b) correct. If not, explain what's going on

4. Sketch the step responses and the frequency responses of circuits with the following pole-zero diagrams. Describe what happens as the poles move around. Draw pictures to illustrate

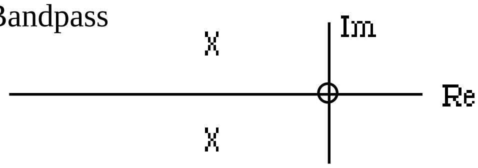
a. Lowpass



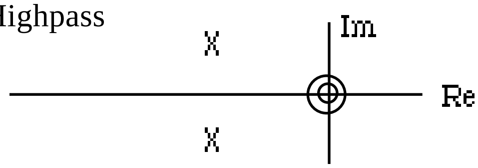
b. Lowpass



c. Bandpass

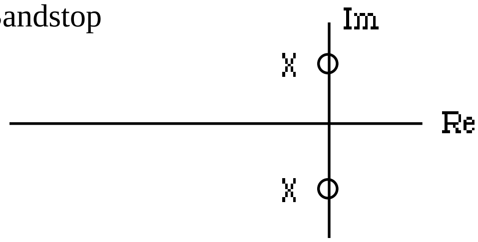


d. Highpass



note that G(s) has two zeros at the origin

e. Bandstop

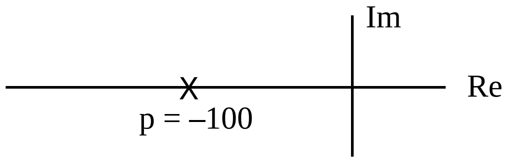


5. Suppose the natural response of a first order low pass circuit decays relatively rapidly. Will it have a narrow or a wide passband. Explain why. Draw pole-zero diagrams and corresponding responses to illustrate

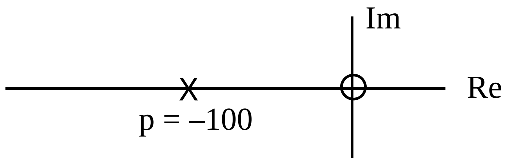
6. How are the 3dB frequencies ω_{3dB} of first order lowpass and highpass circuits related to the magnitudes of the circuit's poles

7. Make use of your results in Problem (6) to sketch the Bode Plots of the transfer functions with pole-zero diagrams as follows

a.



b.



8. Show that when ω is large then the magnitude of

$$G(j\omega) = \frac{K}{j\omega - p} \approx \frac{K}{j\omega}$$

is decreasing at a rate of 20 dB/decade – which means that every time ω increases by a factor of ten, the magnitude |G(jω)| will decrease by 20 dB

9. By how many dB/decade are the magnitudes of the following transfer functions changing when ω is large. Explain how you got your results

   a. $G(j\omega) = \dfrac{K}{(j\omega)^2 - 1000\,j\omega + 10^6}$

   b. $G(j\omega) = \dfrac{K\,j\omega}{(j\omega)^2 - 1000\,j\omega + 10^6}$

10. By how many dB/decade is the magnitude of a transfer function G(jω) with m zeros and n poles changing when ω is large. Explain how you got your result