ECE 307 - POLES AND ZEROS - INVESTIGATION 20
POLES AND ZEROS AND FREQUENCY RESPONSES - PART I

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To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

Up to now we've been focusing on transfer function poles. As we've seen, they determine the shape of the natural response and the time it takes for them to decay. We now turn our attention to the frequency response. We know from ECE 209 how to do the algebra to calculate and then sketch $|G(j\omega)|$ for a circuit with transfer function $G(j\omega)$. The objective of this investigation is to find the geometric relation between the locations of the poles and the **zeros** - the roots $z_1, z_2, \ldots, z_m$ of the numerator of G(s) - and the frequency response.

1. Pulling all our results together we see that general transfer functions G(s) with zeros $z_1, z_2, \ldots, z_m$ and poles $p_1, p_2, \ldots, p_n$ can be written as follows

$$G(s) = K\frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)}$$

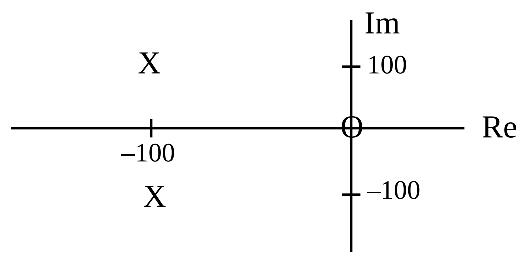
   where the constant K depends on the circuit. Given all this

   a. Find the poles and zeros of the following tranfer function

$$G(s) = K\frac{(s+j100)(s-j100)}{(s+100-j1000)(s+100+j1000)}$$

   and then plot them in a pole-zero diagram. Note that we use O's to show the locations of the zeros.

   b. Find G(s) with the following pole zero diagram



   to within a constant K

2. Now suppose

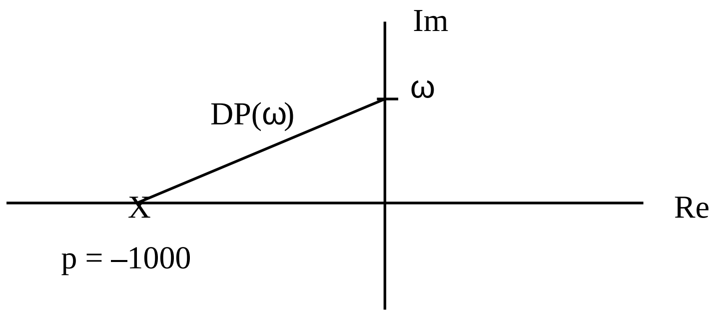
$$G(s) = \frac{1000}{s+1000} = \frac{1000}{s-p}$$

   a. Sketch the frequency response with ω on a log scale starting at $\omega = 1$

$$|G(j\omega)| = \frac{1000}{|j\omega + 1000|}$$

Describe what's happening as ω increases

b. Now verify that the term in the denominator $DP(\omega) = |j\omega + 1000|$ is the distance in the complex plane between the pole $p = -1000$ and the complex number $j\omega$ with imaginary part ω as indicated in the following pole-zero diagram



c. What happens to the distance $DP(\omega)$ as ω increases. Draw pictures to illustrate what's happening to $DP(\omega)$ for several values of ω. How does this correlate with what's happening to $|G(j\omega)|$ as ω increases

d. Express $|G(j\omega)|$ as a function of the distance $DP(\omega)$

e. Find an RC circuit with this pole-zero diagram

3. Generalizing on the results from the previous section, let us consider

$$G(s) = \frac{s}{s+1000}$$

a. Sketch the frequency response with ω on a log scale starting at $\omega = 1$

$$|G(j\omega)| = \frac{|j\omega|}{|j\omega + 1000|}$$

Describe what's going on

b. Draw a pole-zero diagram for G(s) and label the distances $DZ(\omega) = |j\omega|$ and $DP(\omega) = |j\omega + 1000|$ for a given ω. State in words what these distances are

c. What happens to the distances $DZ(\omega)$ and $DP(\omega)$ as ω increases

d. Express $|G(j\omega)|$ as a function of $DZ(\omega)$ and $DP(\omega)$

e. Find an RC circuit with this pole-zero diagram

4. And finally for

$$G(s) = \frac{(s + j100)(s - j100)}{(s+100)(s+100-j1000)(s+100+j1000)}$$

a. Find the frequency response $G(j\omega)$

b. Use the following results for complex numbers

$$|z_1 z_2| = |z_1||z_2| \qquad \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$$

to express the magnitude of your transfer function $|G(j\omega)|$ as a function of magnitudes like

$$|j\omega + j100| \quad \text{and} \quad |j\omega + 100 - j1000|$$

c. Now use Pythagoras to show that each such magnitude

$|j\omega - a - jb|$ = the distance in the complex plane between the complex number with imaginary part ω and the complex number $z = a + jb$

d. Use the result in part (c) to identify the distances DZ1(ω), DZ2(ω), DP1(ω), DP2(ω) and DP3(ω) between the complex number with imaginary part ω and the poles and zeros in your expression for $|G(j\omega)|$

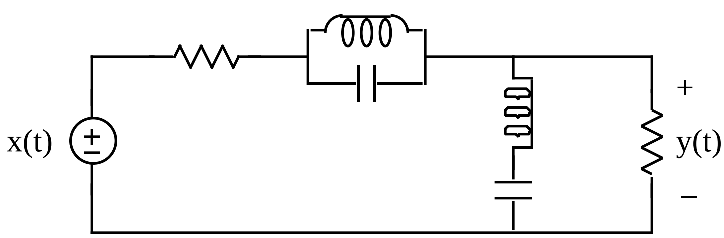
$$|G(j\omega)| = K \frac{DZ1(\omega) \cdot DZ2(\omega)}{DP1(\omega) \cdot DP2(\omega) \cdot DP3(\omega)}$$

Memorize this expression

e. Draw a pole-zero diagram and label the distances DZ1(ω), DZ2(ω), DP1(ω), DP2(ω) and DP3(ω) when $\omega = 1500$

f. Describe how $|G(j\omega)|$ depends on the distances DZ(ω) and DP(ω)

5. Sketch the zeros of the following circuit



on its pole-zero diagram. Explain how you got your result and then sketch $|G(j\omega)|$. Hint - find y(t) at the resonance frequencies ω_{01} and ω_{02} of the series and parallel LC circuits.