ECE 307 - COMPLEX FOURIER SERIES - INVESTIGATION 2
CALCULATION OF COMPLEX FOURIER COEFFICIENTS

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To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

We saw in the previous investigation that Fourier Series Expansions of real periodic signals x(t) like the following

$$x(t) = c_o + \sum_{k=1}^{\infty} c_k \cos(k\omega_o t + \theta_k)$$

can be expressed as sums of complex exponentials as follows

$$x(t) = \sum_{k=-\infty}^{\infty} X_k \, e^{jk\omega_o t}$$

with terms containing not only the imaginary number j but also the **negative frequencies** $-\omega_o$, $-2\omega_o$, · · · . We've come a long, long way. **Memorize** this expression for x(t).

The objective of this investigation is to come up with integral equations for calculating the complex Fourier coefficients X_k. As you might guess, we're going to be using basically the same kinds of mathematical tricks we used in ECE 209 to calculate a_o and the a_k's and b_k's

Let us begin with the following periodic signal x(t) equal to the finite sum

$$x(t) = X_{-2}e^{-j200t} + X_{-1}e^{-j100t} + X_0 + X_1e^{j100t} + X_2e^{j200t}$$

Our goal is to come up with expressions for the complex Fourier coefficients X_{-2}, X_{-1}, X_0, X_1 and X_2 in terms of x(t)

1. First show that

$$x(t) = X_{-2}e^{-j200t} + X_{-1}e^{-j100t} + X_0 + X_1e^{j100t} + X_2e^{j200t}$$

   as given above is periodic with period T = 2π/100. Show, in particular, that x(t) = x(t + 2π/100) is true for all time t

2. The objective of this problem is to derive the math identity we're going to use in developing our equations for the X_k's
   a. First show that the integral of the periodic signal

$$e^{j100t}$$

   is zero over a period T = 2π/100 (Note that it will be easier to calculate the integral if you **don't** convert the complex exponential to sines and cosines)
   b. Now show that the integral of

$$e^{jm100t}$$

is also zero over the time $T = 2\pi/100$ for any integer $m \neq 0$

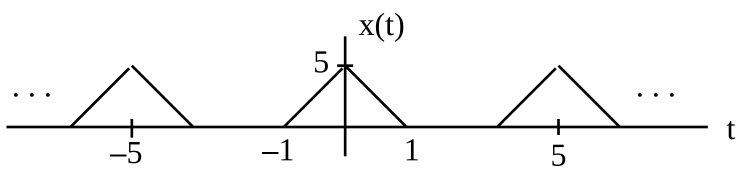
c. Explain in words why these integrals are zero. Hint – make use of the fact that the integral of a sinusoid over an integer number of periods is zero

3. Use your results from Problem (2) to
   a. Find an expression for X_0 in terms of x(t) by taking the integral of both sides over the period $T = 2\pi/100$. Describe what happens to all the terms when you take the integral
   b. Find an expression for X_1 by multiplying both sides of x(t) by e^{-j100t} and then again integrating over the period $T = 2\pi/100$
   c. Then do analogous multiplying and integrating to come up with expressions for X_{-2}, X_{-1} and X_2

4. Generalize on your results in Problem (3) to obtain an expression for the general coefficient X_k in the complex exponential Fourier Series expression

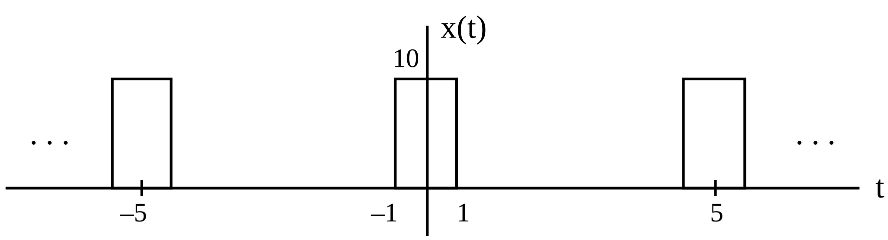
$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{jk\omega_o t}$$

5. Given the following periodic signal



   a. Find X_o
   b. What is the "physical" meaning of X_o

6. Set up the equations for calculating X_o and the X_k's for the following pulse train



It's not necessary to carry through the calculations of the integrals. We'll do that in the next investigation

7. Note that we write our X_k's in the standard form $re^{j\theta}$ with $r > 0$
   a. Convert $X_3 = -2$ to standard form
   b. Convert $X_3 = -2e^{j\pi/3}$ to standard form

8. Show that $e^{j5\pi/2} = e^{j\pi/2}$