ECE 307 - POLES AND ZEROS - INVESTIGATION 19
POLES AND THE NATURAL RESPONSE - PART II

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To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

As we've already seen, the poles of a transfer function G(s) - the roots of its denominator - determine what the natural response looks like. The objective of this investigation is to see what happens to the natural response as the poles move around

1. First for some drill –
   a. Find the poles p_1 and p_2 of the following transfer function

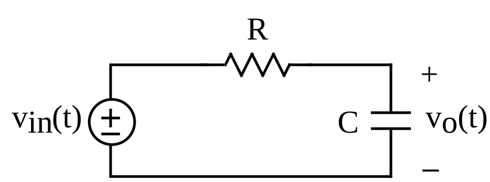
$$G(s) = \frac{100s}{s^2 + 100s + 10^4} = \frac{100s}{(s - p_1)(s - p_2)}$$

   b. Find the denominator polynomial D(s) of a transfer function G(s) with poles

$$p_1 = 0 \qquad p_2, p_2^* = -100 \pm j\,200$$

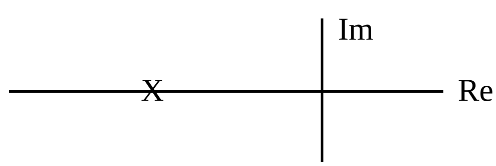
   c. Write out a general expression for the terms of the partial fraction expansion for the poles in part (b)

2. Find the time constant τ and the pole p of the following RC circuit

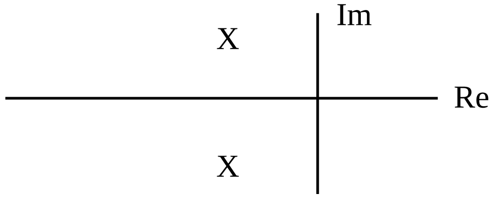


   How are τ and p related. What does a large time constant τ imply about the magnitude of the pole p. **Memorize** this relation.

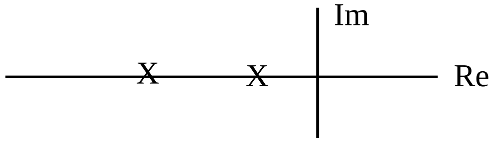
3. One particularly nice thing to do with poles - as it turns out - is to plot them in the complex plane with X's as shown below. We refer to these graphs as **pole-zero diagrams** (We'll introduce zeros in the next investigation - they're simply the roots of the numerator of G(s))
   a. Verify that a circuit with a natural response $y_n(t) = Ke^{-at}$ with $a > 0$ has its pole on the negative real axis of the complex plane as follows



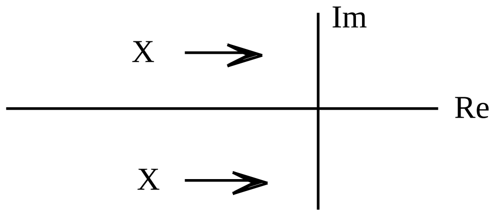
   b. Verify that a second order underdamped circuit with a natural response $y_n(t) = Ke^{-at} \cos(bt + \theta)$ with $a > 0$ has its poles in the complex plane as follows



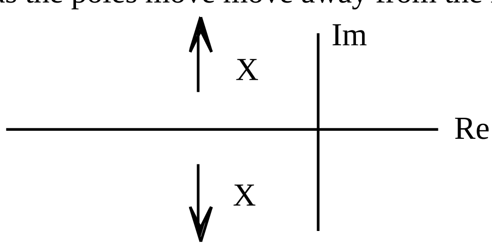
c. Verify that a second order overdamped circuit with a natural response $y_n(t) = K_1e^{-at} + K_2e^{-bt}$ with $a > 0$ and $b > 0$ has its poles in the complex plane as follows



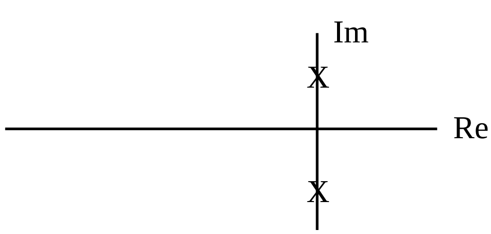
4. Given the following pole-zero diagrams
   a. Draw a **sequence of graphs** to illustrate what happens to the natural response of the corresponding circuit as the poles move toward the imaginary axis as follows



   b. Draw a **sequence of graphs** to illustrate what happens to the natural response of the corresponding circuit as the poles move move away from the real axis as follows

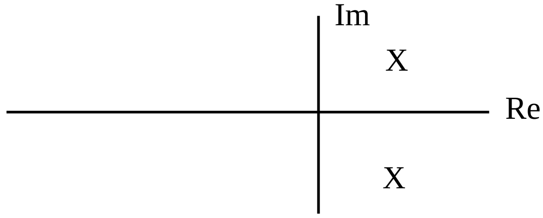


5. Sketch the natural response of a circuit with poles on the imaginary axis as follows



Describe your sketch. What's an example of a circuit that has poles on the imaginary axis

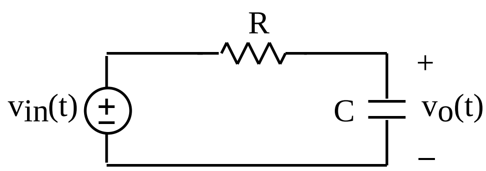
6. Sketch the natural response of a circuit with poles in the right half of the complex plane as follows



Describe your sketch

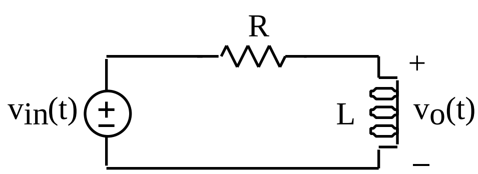
7. How is a pole's distance from the imaginary axis related to the time it takes the corresponding term of the natural response to decay to zero - assuming, of course, that the pole is in the left half of the complex plane

8. How does increasing the value of R in the following circuit



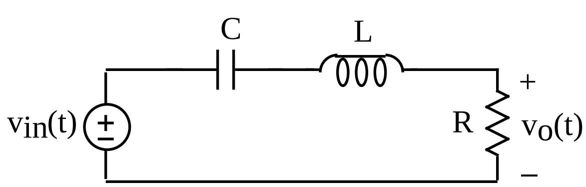
affect the location of the pole. Why. Do an example to verify your answer

9. How does increasing the value of R in the following circuit



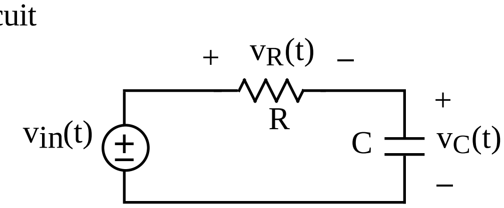
affect the location of the pole. Why. Do an example to verify your answer

10. How does increasing the value of R in the following circuit

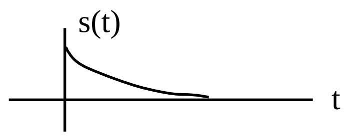


affect the location of the pole. Why. Illustrate with a pole-zero diagram

11. Given the following circuit



with step response s(t) as follows



a. Is s(t) the step response of the capacitor voltage or resistor voltage. Explain in words how you can tell.
b. How will increasing the magnitude of the pole affect s(t)

12. How would you go about finding the pole of a first order circuit like the following