ECE 307 - POLES AND ZEROS - INVESTIGATION 18
POLES AND NATURAL RESPONSES - PART I

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To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

Transfer functions G(s) are really great. All we have to do to find the **zero state response** of a circuit – the response when all the initial conditions are zero – is take the inverse LaPlace Transform of

$$Y(s) = G(s)\,X(s)$$

The objective of this investigation is to find the relation between the natural part of a circuit's response and the poles of G(s) - the roots of the denominator of G(s).

1. Let us begin by doing an example. Suppose G(s) as follows

$$G(s) = \frac{Y(s)}{X(s)} = \frac{10^9}{(s + 100 - j1000)(s + 100 + j1000)}$$

   is the transfer function of a given circuit
   a. Do the partial fraction expansion for finding the **step response** of the circuit – the response when the input x(t) is the unit step u(t) and all initial conditions are zero.
   b. Identify those terms of the partial fraction expansion corresponding to the natural response and those to the forced response
   c. Is the circuit underdamped, overdamped or critically damped. How can you tell
   d. Sketch the natural and forced responses of the circuit
   e. Sketch the complete response

2. Write out an expression for the terms in a partial fraction expansion of the zero state response Y(s) of a circuit with transfer function

$$G(s) = \frac{10^9 s}{(s + 100)(s + 500 - j1000)(s + 500 + j1000)}$$

   and input x(t) = cos (1000t) u(t). It's not necessary to actually calculate the K's. Identify those terms that determine the natural part and those terms that determine the forced parts of the response.

3. As we see in Problems (1) and (2), the natural part of the response of a circuit comes from the roots of the denominator of G(s). We refer to these roots as the **poles** $p_1, p_2, \ldots, p_n$ of G(s) and in general write

$$G(s) = \frac{N(s)}{(s - p_1)(s - p_2)\cdots(s - p_n)}$$

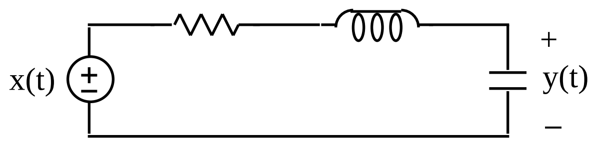
   where N(s) is the numerator polynomial.
   a. Find the poles of the transfer function in Problem (2)
   b. Verify that there is a term in the partial fraction expansion of Y(s) = G(s) X(s) for each pole
   c. Make use of your result in part (b) to explain the relationship between the poles of G(s)

and the terms in the natural part of the response of y(t)

4. Sketch the natural response $y_n(t)$ of a circuit with poles
   a. $p_1 = -100$
   b. $p_1 = -100$, $p_2 = -200$
   c. $p_1 = -100 + j200$, $p_2 = p_1^*$

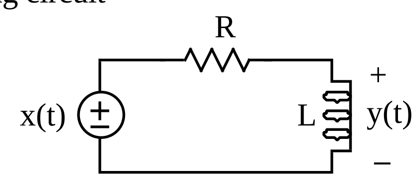
5. What are the poles of circuits with natural responses
   a. $y_n(t) = 10e^{-100t}$
   b. $y_n(t) = 5e^{-200t} + 3e^{-400t}$
   c. $y_n(t) = 5\,e^{-100t}\cos(300t + 1.2)$

6. Given the following 2nd order RLC circuit



   a. Sketch the step response if the circuit is underdamped. Come up with a possible set of poles for your circuit
   b. Sketch the step response if the circuit is overdamped. Come up with a possible set of poles for your circuit

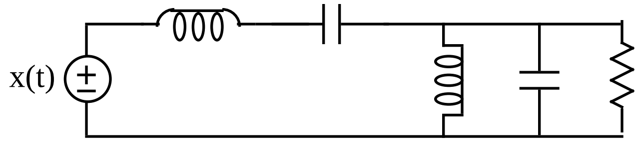
7. Find the pole of the following circuit



8. Given that a circuit with n poles $p_1, p_2, \ldots, p_n$ has a transfer function as follows

$$G(s) = \frac{N(s)}{(s - p_1)(s - p_2)\cdots(s - p_n)}$$

How large is n if the circuit contains two capacitors (not in series or parallel) and two inductors (not in series or parallel) like the following



Explain how you got your answer