ECE 307 - LAPLACE TRANSFORM - INVESTIGATION 16
BILATERAL LAPLACE TRANSFORMED NETWORKS

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To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

The objective of this investigation is to develop bilateral LaPlace Transformed circuits analogous to the Fourier and phasor circuits we've developed. This will enable us to obtain the LaPlace Transforms of circuit variables without having to first write a circuit's differential equation.

1. We know from our previous investigations that the Fourier Transforms of the voltages across and currents through resistors, capacitors and inductors are related by

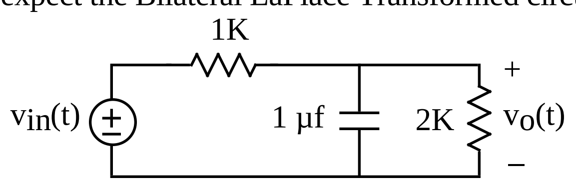
$$V_R(\omega) = R\, I_R(\omega) \qquad V_L(\omega) = j\omega L\, I_L(\omega) \qquad V_C(\omega) = \frac{1}{j\omega C} I_C(\omega)$$

   a. How would you expect that the Bilateral LaPlaceTransforms of the voltages V(s) and currents I(s) of resistors, capacitors and inductors are related. What makes you think so. Set up a Table of your conjectures
   b. Now derive the relationships between the Bilateral LaPlace Transforms of the voltages and currents of resistors, capacitors and inductors from the time relations

$$v(t) = Ri(t) \qquad i_C(t) = C\frac{dv_C(t)}{dt} \qquad v_L(t) = L\frac{di_L(t)}{dt}$$

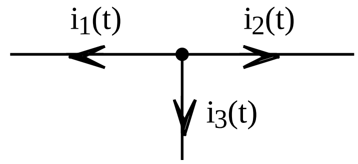
   c. Were your conjectures in part (a) correct. If not, explain what in fact is going on

2. Draw what you would expect the Bilateral LaPlace Transformed circuit of the following circuit

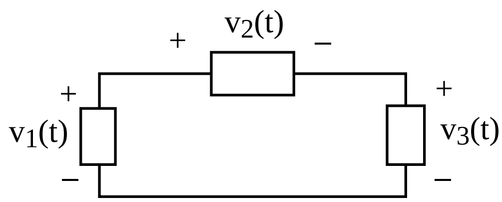


   to look like. Explain how you got your result. Then draw the corresponding Fourier Transform and phasor circuits. What's the relationship between these circuits

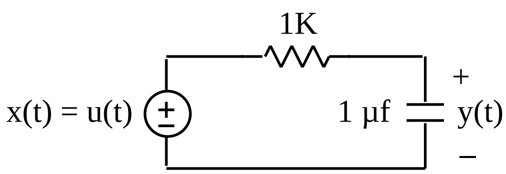
3. The objective of this and the next problem is to demonstrate that Kirchhoff's Laws hold for LaPlace Transformed circuits. First show that the LaPlace Transforms of the currents at the following node satisfy Kirchhoff's Current Law



4. Now show that the LaPlace Transforms of the voltages around the closed loop in the following circuit satisfy Kirchhoff's Voltage Law



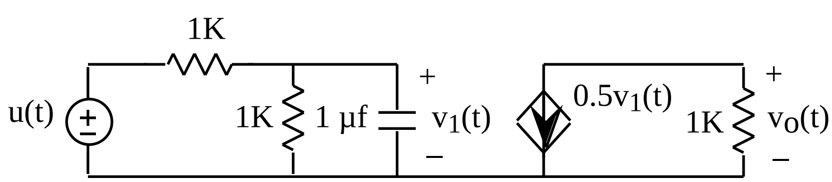
5. Generalizing on the results of Problems (3) and (4) it can be shown that Kirchhoff's Current Law is satisfied at all nodes of LaPlace Transformed circuits and Kirchhoff's Voltage Law is satisfied around all closed loops. As a result we can use all the analysis methods based on Kirchhoff's Laws that we learned in ECE 109 – everything from voltage division to node equations – to analyze LaPlace Transformed circuits. Verify that voltage division and node equations give the same result for $Y_o(s)$ in the following circuit



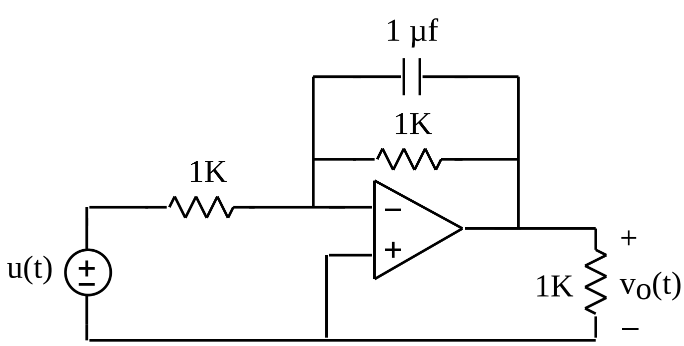
with zero initial conditions

6. Find $v_o(t)$ in each of the following circuits with zero initial conditions

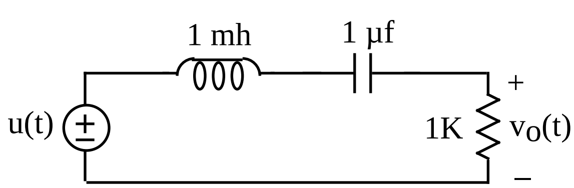
a.



b.



7. Find $V_o(s)$ for the following circuit with zero initial conditions



Then sketch what you expect $v_o(t)$ to look like. Is the natural response underdamped, overdamped or ciritcally damped. How can you tell