ECE 307 - FOURIER TRANSFORMS - INVESTIGATION 11
USING FOURIER TRANSFORMS IN CIRCUIT ANALYSIS

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To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

The objective of this investigation is to show how we can make use of Fourier Transforms to analyze linear circuits. Our results will be very much like those we developed for analyzing phasor circuits

1. First show that

$$F\left[\frac{dx(t)}{dt}\right] = j\omega\, X(\omega)$$

   Hint – Calculate the Fourier Transform by integrating by parts. Make use of the fact that x(t) – or at least its truncated version – satisfies $x(-\infty) = x(\infty) = 0$. How is this result similar to the corresponding result for phasors

2. Show that

$$F[a_1x_1(t) + a_2x_2(t)] = a_1X_1(\omega) + a_2X_2(\omega)$$

3. Now apply your results from Problems (1) and (2) to solve for y(t) satisfying the following differential equation

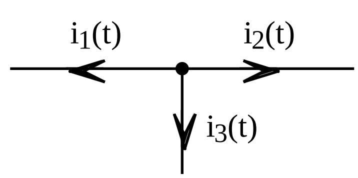
$$y' + 1000y = 500\,\delta(t)$$

   by first taking the Fourier Transform of both sides of the equation, then solving the result for $Y(\omega)$ and finally making use of your results in the last investigation on Fourier Transforms of exponentials to find y(t)

4. Finding $Y(\omega)$ like we did in Problem (3) is easy enough if we have the differential equation for y(t). But as we all know, finding the differential equation for a general circuit can be very tedious. To get around this problem we pursue a path analogous to what we did with phasors to come up with Fourier Transformed circuits
   a. First show that the Fourier Transforms of the voltages and currents of resistors, capacitors and inductors are related as follows

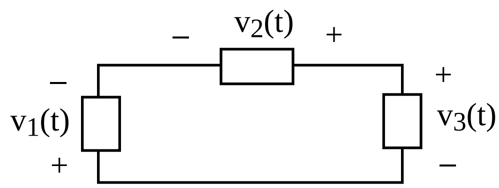
$$V_R(\omega) = RI_R(\omega) \qquad V_C(\omega) = \frac{1}{j\omega C} I_C(\omega) \qquad V_L(\omega) = j\omega\, LI_L(\omega)$$

   b. Illustrate the fact that the Fourier Transforms of currents satisfy Kirchhoff's Current Law by showing that the Fourier Transforms of the following currents



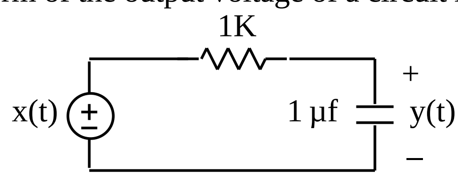
satisfy $I_1(\omega) + I_2(\omega) + I_3(\omega) = 0$.

c. Then illustrate the fact that the Fourier Transforms of voltages satisfy Kirchhoff's Voltage Law by showing that the Fourier Transforms of the following voltages

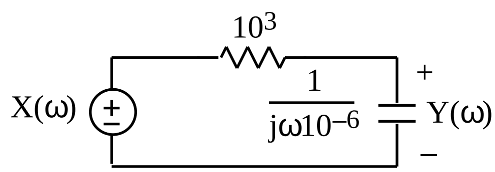


satisfy $V_1(\omega) - V_2(\omega) + V_3(\omega) = 0$

d. And finally make use of your results in parts (a), (b) and (c) to justify the fact that we can find the Fourier Transform of the output voltage of a circuit like the following



by analyzing the corresponding **Fourier Transformed Circuit**

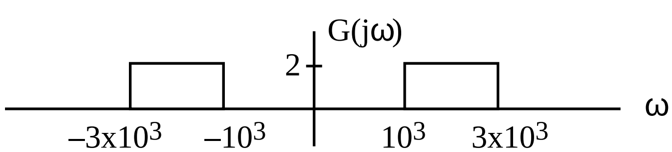


just like we analyze phasor circuits

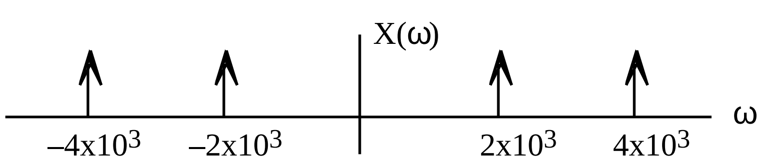
e. Make use of your result in part (d) to find $v_o(t)$ if $x(t) = \delta(t)$

5. Find the expression relating $Y(\omega)$, $X(\omega)$ and $G(j\omega)$ of a linear circuit where $Y(\omega)$ is equal to the Fourier Transform of the output, $X(\omega)$ is equal to the Fourier Transform of the input and $G(j\omega)$ is the transfer function. **Memorize** this result

6. Given a linear circuit with transfer function $G(j\omega)$ as follows



and input x(t) with Fourier Transform as follows



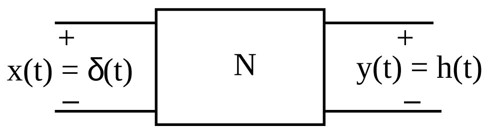
where each impulse is of magnitude one

a. Sketch the Fourier Transform Y(ω) of the output
b. Find y(t)

7. Make use of the relationship

$$Y(\omega) = G(j\omega)\,X(\omega)$$

to find the Fourier Transform H(ω) of the impulse response h(t) of a linear circuit as follows



as a function of G(jω). This result is fundamental. **Memorize** it.

8. Find the Fourier Transform H(ω) of the impulse response h(t) of a circuit with transfer function G(jω) as follows

$$G(j\omega) = \frac{400\, j\omega}{(j\omega)^2 + 500\, j\omega + 10^6}$$

Now finding H(ω) for this circuit was pretty easy. Finding the inverse Fourier Transform h(t) is harder. One good way is to use numerical methods as developed in digital signal processing classes. We will develop an analytical method for finding these responses when we do LaPlace Transforms in the next set of investigations