ECE 306 - DISCRETE CONVOLUTION - INVESTIGATION 7
DISCRETE CONVOLUTION - PART II

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To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

We know from the last investigation that the zero state responses of linear time-invariant causal discrete systems to arbitrary inputs x[n] as follows

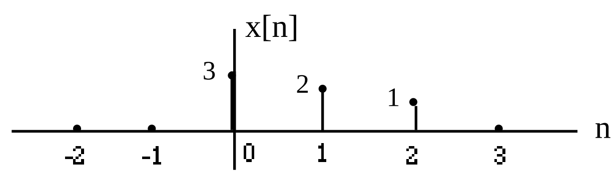
$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

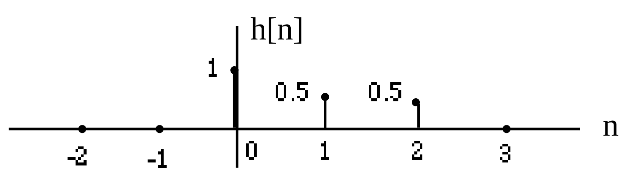
can be expressed as sums of impulse responses - as convolution sums - as follows

$$y[n] = \sum_{k=0}^{n} x[k]h[n-k]$$

if $x[n] = 0$ for $x < 0$. The objective of this investigation is to illustrate the graphical interpretation of convolution sums as well as demonstrate some basic properties of convolution.

1. The objective of this first problem is to review the calculation of a convolution sum. Given a linear time-invariant discrete with input x[n] and impulse response $h[n]$ as follows





   Find and plot the zero state response to $y[n]$ by plotting and then adding up the responses to $3\delta[n]$, $2\delta[n-1]$ and $\delta[n-2]$

2. In Problem (1) we illustrated how to calculate a convolution sum of the form

$$y[n] = \sum_{k=0}^{n} x[k]h[n-k]$$

   by plotting and then adding up the responses $x[0]h[n]$, $x[1]h[n-1]$, . . . , $x[n]h[0]$. Alternatively we can find $y[n]$ for any particular n like $n = 2$

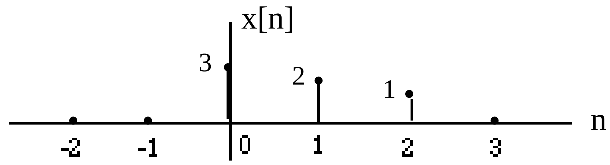
$$y[2] = \sum_{k=0}^{n} x[k]h[2-k]$$

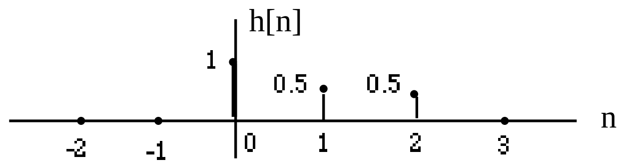
   as follows

(1) Plot $x[k]$ as a function of k
(2) Plot $h[2-k]$ as a function of k
(3) Add up the products of the corresponding terms

We refer to this procedure as **graphical convolution**. **Memorize** it.

a. Use graphical convolution to find and plot $y[0]$, $y[1]$, . . . , $y[5]$ for the same linear time-invariant discrete system as in Problem (1) as follows





b. Put your results from part (a) above and from Problem (1) in a Table. Verify that they're the same

3. Given a linear time-invariant discrete system with input $x[n]$ and impulse response $h[n]$ as follows

$$x[0]=2\,,\quad x[1]=-3\,,\quad x[2]=1$$
$$h[0]=2\,,\quad h[1]=1\,,\quad h[2]=0\,,\quad h[3]=0\,,\quad . . .$$

a. Find the difference equation for $y[n]$
b. Make use of your difference equation in part (a) to directly calculate the zero state response to $x[n]$
c. Use graphical convolution to find the zero state response $y[n]$
d. Verify that your results in parts (b) and (c) are the same. Use a Table

4. Use graphical convolution to find the zero state response $y[n]$ at $n=0$, 1, 2 and 3 of the linear time-invariant discrete system with input $x[n]=u[n]$ and impulse response $h[n]=0.5^n u[n]$ at $n=$ 0, 1, 2 and 3. Describe in words how you got your results.

5. Verify that the following two sums give the same result for $y[2]$

$$y[2]=\sum_{k=0}^{2} x[k]h[2-k] \qquad \text{and} \qquad y[2]=\sum_{k=0}^{2} h[k]x[2-k]$$

for linear time-invariant discrete systems.

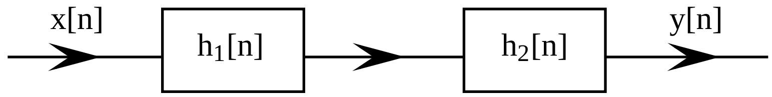
6. Generalize on your result in Problem (5) to verify that in general

$$y[n]=\sum_{k=0}^{n} x[k]h[n-k]=\sum_{k=0}^{n} h[k]x[n-k]$$

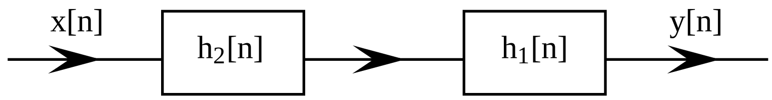
for linear time-invariant causal systems with inputs starting at $n=0$. Hint - write out the terms in the two sums. **Memorize** this result.

7. Now make use of convolution to show that the zero state response of the following discrete

system



is equal to that of



with h_1 and h_2 reversed. Hint - make use of the following two relations

(1) $v_1[n] \quad v_2[n] = v_2[n] \quad v_1[n]$

(2) $v_1[n] \quad \left(v_2[n] \quad v_3[n]\right) = \left(v_1[n] \quad v_2[n]\right) \quad v_3[n]$

to show that

$$h_2[n] \quad \left(h_1[n] \quad x[n]\right) = h_1[n] \quad \left(h_2[n] \quad x[n]\right)$$

8. Math Review: Express $X = \dfrac{5}{1-0.5e^{-j2}}$ as a complex exponential of the form $re^{j\theta}$

9. Math Review: Express the following sum of complex exponentials as a sum of sinusoids if $X_0 = 1, \quad X_1 = 3e^{j0.8}, \quad X_{-1} = X_1, \quad X_2 = 2e^{-j1.2}, \quad X_{-2} = X_2$

$$\sum_{k=-2}^{2} X_k e^{j2\pi k1000t}$$

10. MATLAB - Copy, save and then run the following M-file

```
A = 2;  ph = 1.2;  f = 1000;  T = 1/f;
t = linspace(0, 3*T, 100);
x = A*cos (2*pi*f*t + ph);
y = 2*A*cos (2*pi*f*t + ph);
plot (t, x, t, y)
grid;
title ('cosines'); xlabel ('t');  ylabel ('x(t), y(t)');
```

Then use your results to describe the syntax for plotting two functions on the same graph

11. MATLAB - Copy, save and then run the following M-file

```
A = 2;  ph = 1.2;  f = 1000;  fs = 10000;  Ts = 1/fs;
n = 0: 10;
x = A * cos (2*pi*f*n*Ts + ph);
stem (n,x)
grid;
title ('sampled cosine'); xlabel ('n');  ylabel ('x[n]');
```

Then use your results to

a. Explain what this program is doing
b. What does the instruction "stem" do