ECE 306 - ONE-SIDED Z-TRANSFORM - INVEST 25
INTRODUCTION TO THE ONE-SIDED Z-TRANSFORM

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From our previous Investigations we know that if we have a difference equation like the following

$$y[n] = 0.5y[n-1] + x[n]$$

then

(1) Poles-zero diagrams are great for sketching frequency and natural responses

(2) Transfer functions $H\left(e^{j2\pi fT_s}\right)$ are the easiest way to calculate sinusoidal steady state responses

(3) Direct evaluation of the difference equation is the easiest way to calculate the values of $y[0],\ y[1],\ y[2],\ \cdots$

(4) The z-transform is a convenient way to get a closed form expression for $y[n]$

The main objective of this Investigation is to show how the **one-sided or unilateral z-transform** can be used to obtain closed form solutions of difference equations with nonzero initial conditions

1. The one-sided z-transform defined as follows

$$\sum_{n=0}^{\infty} x[n]\, z^{-n}$$

is basically the same as the bilateral z-transform except that the sum starts at n = 0 rather than at n = -∞. As a result we have to recalculate the z-transforms of delayed signals but do not have to do any recalculations, however, for signals like

$$u[n],\quad \delta[n],\quad \cos(2\pi f nT)u[n]\ \cdot\cdot\cdot$$

since they are zero for n < 0.

   a. Show that $Z\left[x[n-1]\right] = z^{-1}X(z) + x[-1]$

   b. Show that $Z\left[x[n-2]\right] = z^{-2}X(z) + z^{-1}x[-1] + x[-2]$

   c. Generalize on your results in parts (a) and (b) to find an expression for $Z\left[x[n-k]\right]$

2. Find an expression for $y[n]$ satisfying the following difference equation. Check your answers with those from the difference equation for $y[0]$, $y[1]$ and $y[2]$

$$y[n] = 0.5y[n-1] + u[n] \qquad y[-1] = -1$$

3. Find an expression for $y[n]$ satisfying the following difference equation. Check your answers with those from the difference equation for $y[0]$, $y[1]$ and $y[2]$

$$y[n] = 0.8y[n-2] + 0.5y[n-1] + \delta[n] \qquad y[-2] = 1,\quad y[-1] = -1$$

4. For what initial conditions are the one-sided and bilateral z-transforms of a difference equation equal to each other

5. For what initial conditions will the one-sided z-transform of a LTI difference equation give us

the transfer function $G(z) = Y(z)/X(z)$

6. Find the steady state response of $y[n] = 0.5y[n-1] + 3\cos(0.5n)u[n] \qquad y[-1] = 1$

7. What are the tradeoffs between recursive and nonrecursive digital filters