ECE 306 - THE Z-TRANSFORM - INVESTIGATION 22
TRANSFER FUNCTIONS OF LINEAR DISCRETE SYSTEMS

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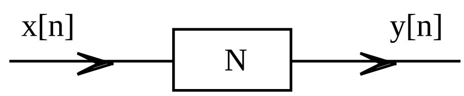
To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

From the last Investigation we know how to use the z-transform to find the zero state responses of linear difference equations with constant coefficients. The objective of this Investigation is to explore some of the basic properties of transfer functions $H(z)$ of linear discrete systems.

1. We begin with a review problem. Find the step response of the difference equation

$$y[n] = 0.5y[n-1] + 2x[n]$$

2. How would you define the transfer function $H(z)$ of a linear discrete system with input x[n] and output $y[n]$ as follows



in terms of the corresponding bilateral z-transforms X(z) and Y(z)

3. Find the transfer functions $H(z) = Y(z)/X(z)$ for linear discrete systems with the following difference equations
   a. $y[n] = 0.5x[n] + 2x[n-1] - x[n-2]$
   b. $y[n] = 0.5y[n-1] + x[n] - 0.8x[n-1]$

4. Find the difference equations having transfer functions
   a. $H(z) = \dfrac{z}{z+0.8}$
   b. $H(z) = \dfrac{z+0.5}{z+0.8}$

5. What is $Y(z) = H(z)X(z)$ equal to when $x[n] = \delta[n]$

6. From Problem (5) we know that the bilateral z-transform of the impulse response of a linear discrete system is equal to its transfer function H(z). And so

$$H(z) = \sum_{n=-\infty}^{\infty} h[n]z^{-n}$$

where h[n] is the impulse response of the system. **Memorize** this result. Then use it to find the transfer function of linear discrete systems with impulse responses
   a. $h[0] = 1$, $h[1] = -1$, $h[2] = 2$ and $h[n] = 0$ for all other n
   b. $h[n] = 3(0.5)^n u[n]$

7. Find the step response of the linear discrete system with impulse response $h[n] = 3(0.5)^n u[n]$. Hint - first find the transfer function $H(z)$

8. The objective of this problem is to review the relationship between the frequency responses $H\left(e^{j2\pi f T_s}\right)$ and transfer functions $H(z)$ of linear discrete systems. Given the following difference equation

$$y[n] = 0.5y[n-1] + x[n] + 2x[n-1]$$

   a. Verify that the method we used to find $H(z)$ in Investigation 13 is in fact the same as we obtain when we use the z-transform to find it
   b. Make use of your $H(z)$ to find $H\left(e^{j2\pi f T_s}\right)$ like you did in Investigation 13

9. Generalizing on the result of Problem (8) it can be shown that

$$H\left(e^{j2\pi f T_s}\right) = H(z)\Big|_{z=e^{j2\pi f T_s}}$$

   for all transfer functions $H(z)$. **Memorize** this relationship. Then use it to find and sketch the magnitude of the frequency response of the discrete system with transfer function

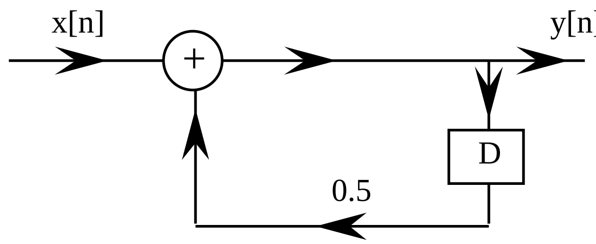
$$H(z) = \frac{z}{z-0.5}$$

10. Find the sinusoidal steady state response y[n] of the discrete system with transfer function

$$H(z) = \frac{z}{z^2 - 0.5z + 1}$$

   to the sinusoid $x(t) = 5\cos(2\pi 1000t + 1.2)$ sampled at $f_s = 5000$ Hz

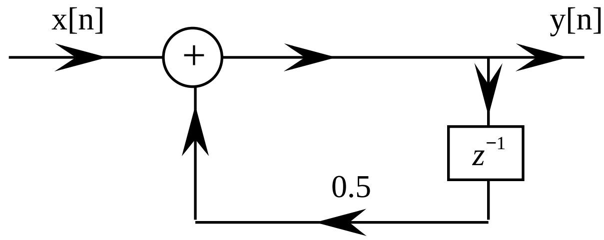
11. When we first introduced discrete systems in Investigation 6 we represented their physical implementation with block diagrams like the following



   where D corresponds to the memory - the D flip-flops - for storing previous values of the $x[n]$'s and $y[n]$'s. Now that we've introduced the z-transform and derived the relation

$$Z\left[w[n-1]\right] = z^{-1}W(z)$$

   we replace each of the D's in our block diagram with a z^{-1} as follows



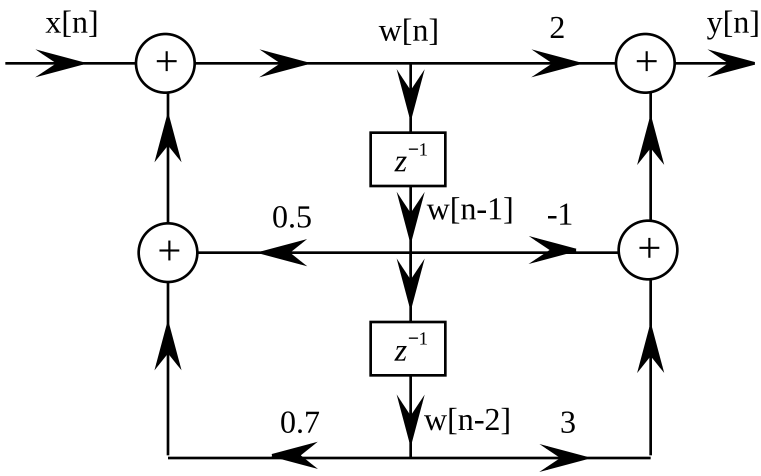
Doing this, as it turns out, facilitates both the design and analysis of these block diagrams

a. Draw a block diagram realization that uses z^{-1} for the delay for the following difference equation

$$y[n] = 2x[n] - 3x[n-1]$$

b. Repeat part (a) for $y[n] = -0.8y[n-1] + 2x[n] + 3x[n-1]$

12. Given the following Direct Form II Realization of a second order difference equation



a. Find $w[n]$ as a function of $x[n]$ and $w[n-1]$ and $w[n-2]$
b. Find $W(z)$ as a function of $X(z)$
c. Find $y[n]$ as a function of $w[n]$ and previous values
d. Find $Y(z)$ as a function of $W(z)$
e. Find $Y(z)$ as a function of $X(z)$
f. Find the transfer function $H(z) = Y(z)/X(z)$
g. Find the difference equation for $y[n]$ in terms of $x[n]$ and previous values