ECE 306 - FOURIER ANALYSIS - INVESTIGATION 16
DISCRETE TIME FOURIER SERIES - PART III

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To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

From the last Investigation we know some of the basic properties of Discrete Time Fourier Series. The main objective of this Investigation is to show how Discrete Time Fourier Series expansions can be used to calculate steady state responses to periodic sequences.

1. The main result we use in calculating the steady state responses of linear time-invariant discrete systems to periodic inputs is the fact that the steady state response to

$$X_k e^{jk(2\pi/N)n} \quad \text{is} \quad Y_k e^{jk(2\pi/N)n} = H\left(e^{jk(2\pi/N)}\right) X_k e^{jk(2\pi/N)n}$$

where $H\left(e^{jk(2\pi/N)}\right) = H(z)\Big|_{z \to e^{jk(2\pi/N)}}$. The objective of this problem is to demonstrate that this result is true for the following difference equation

$$y[n] = 0.5y[n-1] + x[n]$$

   a. First find the transfer function $H\left(e^{jk(2\pi/N)}\right)$
   b. Then substitute $x[n] = \mathbf{X}_k e^{jk(2\pi/N)n}$ and $y[n] = \mathbf{Y}_k e^{jk(2\pi/N)n}$ into the difference equation and solve for Y_k/X_k
   c. Now verify that $Y_k/X_k = H\left(e^{jk(2\pi/N)}\right)$

2. Generalizing on the result in Problem (1) we have that the steady state response of a linear discrete system to a periodic input as follows

$$\mathrm{x[n]} = \sum_{k=0}^{N-1} \mathbf{X}_k e^{jk(2\pi/N)n}$$

is given by

$$\mathrm{y[n]} = \sum_{k=0}^{N-1} \mathbf{Y}_k e^{jk(2\pi/N)n} = \sum_{k=0}^{N-1} H\left(e^{jk(2\pi/N)}\right) \mathbf{X}_k e^{jk(2\pi/N)n}$$

with

$$\mathbf{Y}_k = H\left(e^{jk(2\pi/N)}\right) \mathbf{X}_k$$

Memorize this result. Then make use of it to find and plot the steady state response y[n] of a discrete system with frequency response as follows

$$H\left(e^{jk(2\pi/N)}\right) = \frac{e^{jk(2\pi/N)}}{1 + 2e^{jk(2\pi/N)}}$$

to a periodic x[n] of period $N = 3$ with DTFS coefficients $\mathbf{X}_0 = 2$, $\mathbf{X}_1 = 2e^{j1.2}$, $\mathbf{X}_2 = 2e^{-j1.2}$

3. Find and plot the steady state response of the following discrete system

$$y[n] = 0.3y[n-1] + 0.5x[n]$$

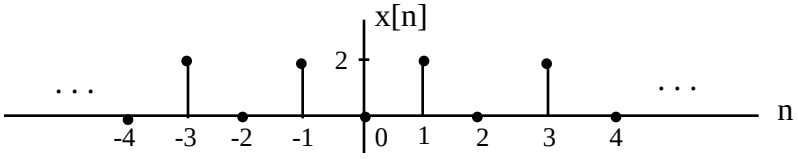
to the periodic input x[n] of period $N = 3$ with DTFS coefficients as follows

$$\mathbf{X}_0 = 2 \qquad \mathbf{X}_1 = 2e^{j1.2} \qquad \mathbf{X}_2 = 2e^{-j1.2}$$

4. Find and plot the steady state response of the following difference equation

$$y[n] = 0.5y[n-1] + x[n]$$

to the following periodic input



5. MATLAB - Copy and run the following M-file

```
x = [1 2];
y = repmat (x, 1, 3)
w = repmat (x, 3, 1)
```

Explain what the instruction *repmat* (replicate matrix) does

6. MATLAB - Copy and run the following M-file

```
m=50;  p=3;
x1 = ones (1, m);  x2 = zeros (1, m);
one_pulse = [x1 x2];
%  Each pulse is 2*m points long
pulse_train = repmat (one_pulse, 1, p);
t = 0 : (2*m)*p - 1;
plot (t, pulse_train)
```

Explain how this program uses the instruction *repmat* (replicate matrix) to plot the pulse train