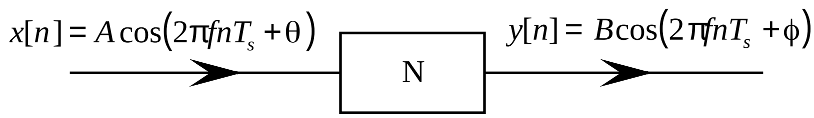
ECE 306 - FREQUENCY RESPONSE - INVESTIGATION 11
FREQUENCY RESPONSES OF DISCRETE SYSTEMS - PART III

FALL 2006 **A.P. FELZER**

From the last Investigation we know how to calculate and make use of frequency responses

$$H\left(e^{j2\pi fT_s}\right)$$

and their graphs to calculate the sinusoidal steady state responses of linear difference equations to sampled sinusoids as illustrated by



The main objectives of this Investigation are to get more practice calculating steady state responses and to show that $H\left(e^{j2\pi fT_s}\right)$ is periodic.

1. We begin with some review problems. Find the transfer function $H\left(e^{j2\pi fT_s}\right)$ for the following linear time-invariant difference equation

$$y[n] = 0.4y[n-1] - 0.8y[n-2] + x[n]$$

2. Find the sinusoidal steady state response $y[n]$ of the difference equation with the following transfer function

$$H\left(e^{j2\pi fT_s}\right) = \frac{e^{j2\pi fT_s}}{1 + 2e^{j2\pi fT_s}}$$

   when the input is $x(t) = 5\cos(2\pi 1000t + 1.2)$ sampled at f_s = 3500 samples/sec

3. Find the steady state response of a discrete system with normalized transfer function as follows

$$H\left(e^{j\Omega}\right) = \frac{e^{j\Omega}}{1 + 3e^{j\Omega}}$$

   to the input $x(t) = 3\cos(2\pi 100t)$ if f_s = 400 samples/sec

4. Now as a rule we need to keep the sampling frequency f_s large enough so that if $x(t)$ is a sinusoid of frequency f then

$$f \le \frac{f_s}{2}$$

   The objective of this problem is to see what happens if f gets larger than $f_s/2$. What's interesting - and actually a little surprising at first - is that frequency responses $H\left(e^{j2\pi fT_s}\right)$ of linear time-invariant discrete systems are periodic. The objective of this problem is to see the underlying reason why this is so.

   a. Use the result in Investigation 2 that the samples of $x(t) = A\cos\left(2\pi ft + \theta\right)$ are the same as $x_m(t) = A\cos\left(2\pi(f + mf_s)t + \theta\right)$ for all integers m to explain why the frequency responses of linear time-invariant difference equations are periodic with period f_s

b. Find the steady state response to the samples of

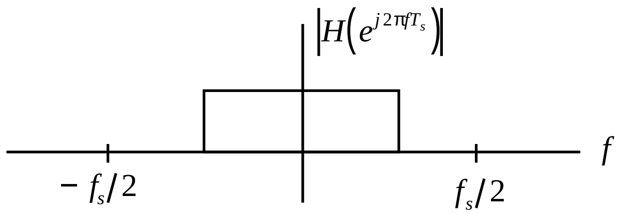
$$x_2(t) = 5\cos\left(2\pi(1000 + 2f_s)t\right)$$

if the sinusoidal steady state response to the samples of $x_2(t) = 5\cos\left(2\pi 1000t\right)$ is $y[n] = 2\cos(3n + 1.2)$. Explain how you got your result

c. Sketch the magnitude $\left|H\left(e^{j2\pi fT_s}\right)\right|$ of the frequency response of a linear time-invariant difference equation for

$$-1.5f_s \le f \le 1.5f_s$$

if $\left|H\left(e^{j2\pi fT_s}\right)\right|$ is as follows in the range $-0.5f_s \le f \le 0.5f_s$



5. Sketch $H\left(e^{j2\pi fT_s}\right)$ of an ideal lowpass filter over the frequency range $-3f_s \le f \le 3f_s$

6. Sketch $H\left(e^{j2\pi fT_s}\right)$ of an ideal highpass filter over the frequency range $-3f_s \le f \le 3f_s$

7. Sketch $H\left(e^{j2\pi fT_s}\right)$ of an ideal bandpass filter over the frequency range $-3f_s \le f \le 3f_s$

8. Use Matlab to obtain a graph of the magnitude of the following transfer function

$$H\left(e^{j2\pi fT_s}\right) = \frac{e^{j2\pi fT_s}}{1 + 2e^{j2\pi fT_s}}$$

as a function of f over at least 3 periods for $T_s = 1$ msec. Note that in Matlab $\left|H\left(e^{j2\pi fT_s}\right)\right| = abs\left(H\left(e^{j2\pi fT_s}\right)\right)$

9. Make use of the fact that we can use superposition to find steady state responses to sums of sinusoids to find the steady state response of a discrete system with the following transfer function

$$H\left(e^{j2\pi fT_s}\right) = \frac{e^{j2\pi fT_s}}{e^{j2\pi fT_s} - 0.5}$$

to $x(t) = 3\cos(2\pi 500t) + 2\cos(2\pi 1000t)$ sampled at $f_s = 2500$ samples/sec

10. Find the steady state response of the difference equation with frequency response

$$H\left(e^{j2\pi fT_s}\right) = \frac{e^{j2\pi fT_s}}{e^{j2\pi fT_s} - 0.5}$$

to the input $x[n]=5$. Hint - a constant is a sinusoid with frequency $f=0$

11. Verify that this transfer function

$$H\left(e^{j2\pi fT_s}\right)=\frac{e^{j2\pi fT_s}}{\left(e^{j2\pi fT_s}+3\right)\left(e^{j2\pi fT_s}+4\right)}$$

satisfies the following general relationship for linear time-invariant discrete systems

$$H\left(e^{-j2\pi fT_s}\right)=H^*\left(e^{j2\pi fT_s}\right)$$

Memorize this result. Hint - make use of the following results

$$\left(z_1 z_2\right)^* = z_1^* z_2^* \qquad \left(\frac{z_1}{z_2}\right)^* = \frac{z_1^*}{z_2^*} \qquad \left(a+be^{j\theta}\right)^* = a+be^{-j\theta}$$

12. Making use of the convolution sum

$$y[n]=\sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

and a little fancy footwork it can be shown that the transfer functions of linear time-invariant discrete systems can be calculated from their impulse responses as follows

$$H\left(e^{j2\pi fT_s}\right)=\sum_{k=-\infty}^{\infty} h[k]e^{-j2\pi kfT_s}$$

Make use of this relation to find the transfer function $H\left(e^{j2\pi fT_s}\right)$ of the discrete system with the following difference equation

$$y[n]=2x[n]-1.5x[n-1]+0.8x[n-2]$$

13. Math Review: Where is $z=e^{j1.2}$ located in the complex plan. Draw a picture to illustrate

14. Math Review: Find the distance between the complex numbers $z_1=e^{j1.2}$ and $z_2=0.5-j0.7$

15. MATLAB - Copy and then save the following M-file as x_intercept.m

```
function  x0 = x_intercept(m,b)
% x_intercept finds the value of x where y=0
% for the line  y = mx + b
x0 = -b/m;
end   % function x_intercept
```

And then run it in the Command Window with

```
x0 = x_intercept(2, -3)
```

How are Matlab functions different from "regular" M-files