

# ECE 306 - THE VERY BASICS - INVESTIGATION 1

## INTRODUCTION TO DISCRETE SIGNALS AND SYSTEMS

FALL 2006

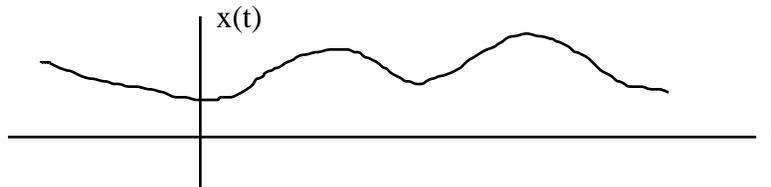
A.P. FELZER

To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

From our previous classes we know how to build *analog* circuits as follows



with circuit elements like resistors, capacitors, diodes, transistors and op amps that can attenuate, amplify, filter, rectify and so on *continuous* inputs  $x(t)$  like the following

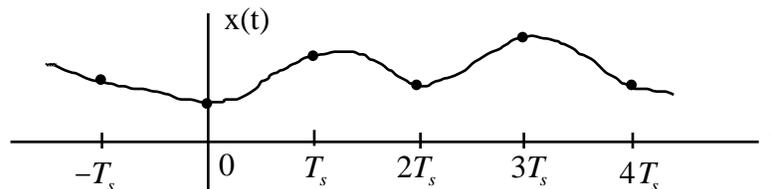


Our main objective in this class is to show how these functions can be implemented with discrete systems of the following form

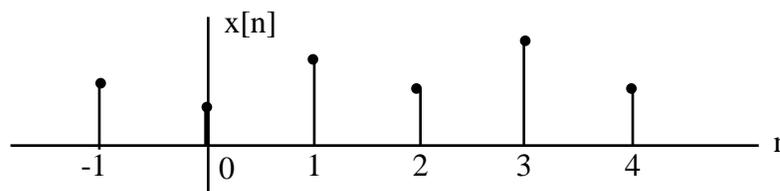


in which

- (1) The analog inputs  $x(t)$  are *sampled* every  $T_s$  seconds as follows

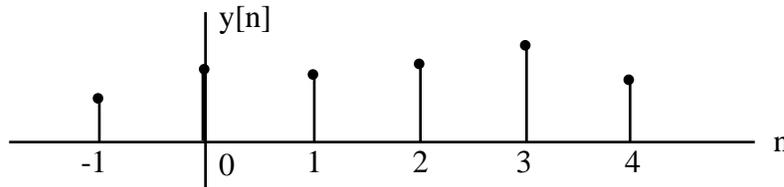


to obtain the *sample values*  $x[n] = x(nT_s)$  as follows



- (2) The sample values - the numbers  $x[n] = x(nT_s)$  - are then processed by a digital circuit

made from gates and flip-flops (really a souped up calculator) that does calculations on the sample values analogous to those done by the analog circuit on the continuous signal  $x(t)$  to give us the following discrete output

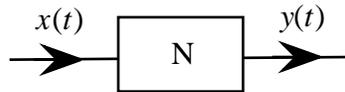


- (3) And finally the discrete outputs  $y[n]$  of the digital circuit are converted back to analog to give us the continuous output  $y(t)$



Our main objective in this first Investigation is to introduce the kinds of equations that characterize discrete systems and show how they're related to the differential equations of analog circuits.

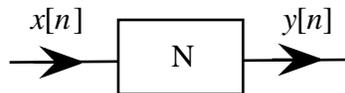
1. We begin with a review problem. Given an analog system  $N$  as follows



with  $x(t) = 5\cos(2000t)$

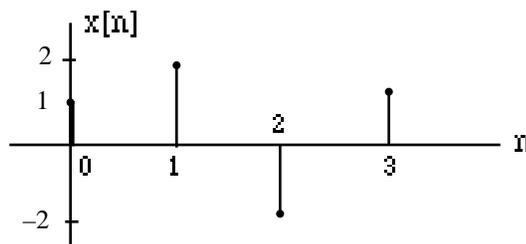
- Sketch  $x(t)$
- Sketch  $y(t)$  if  $N$  is an attenuator with  $y(t) = 0.5x(t)$
- Sketch  $y(t)$  if  $N$  is an amplifier with  $y(t) = 2x(t)$
- Sketch  $y(t)$  if  $N$  is a rectifier with  $y(t) = |x(t)|$

2. Now suppose that  $N$  is a discrete system with input  $x[n]$  and output  $y[n]$  as follows



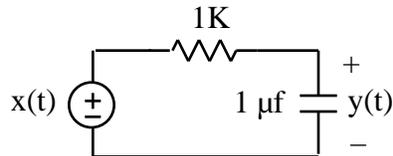
with  $x[-2] = 1, x[-1] = 1, x[0] = -1, x[1] = -2, x[2] = 1, x[3] = 1$

- Sketch a *discrete* plot like the following



of  $x[n]$  as a function of  $n$

- b. Sketch  $y[n]$  as a function of  $n$  if  $N$  is an attenuator with  $y[n] = 0.5x[n]$
  - c. Sketch  $y[n]$  as a function of  $n$  if  $N$  is an amplifier with  $y[n] = 2x[n]$
  - d. Sketch  $y[n]$  as a function of  $n$  if  $N$  is a rectifier with  $y[n] = |x[n]|$
3. From Problem (2) we see that it's pretty straightforward to implement algebraic functions like attenuators, amplifiers and rectifiers with discrete systems. But to design a discrete system that responds to inputs like the following analog RC circuit is a little more involved



One approach is to make use of numerical analysis methods to approximate the circuit's differential equation. As the first step in this process verify that the differential equation for  $y(t)$  is as follows

$$\frac{dy(t)}{dt} + 1000y(t) = 1000x(t)$$

Hint - write the node equation at the output

4. Now the trick to obtaining an equation for a discrete system that responds like an analog RC circuit is to make use of the fact that the voltage  $y(t)$  across the capacitor at time  $t = t_0$  can be written as follows

$$y(t_0) = y(t_0 - t) + t \text{ Average rate at which } y(t) \text{ is changing during the interval } t_0 - t \text{ to } t_0$$

$$y(t_0) = y(t_0 - t) + t \text{ Average of } \frac{dy(t)}{dt} \text{ during } t_0 - t \text{ to } t_0$$

where

$$y(t_0 - t) = \text{the voltage across the capacitor at time } t = t_0 - t$$

- a. Write out in words what the above equation for  $y(t_0)$  is saying
- b. Now as it turns out we can't build digital filters with exactly the same frequency responses as analog filters. But we can build them with responses that are arbitrarily close. The challenge is to come up with a "good" approximation for the average value of the derivative  $\frac{dy(t)}{dt}$  in the interval  $t_0 - t$  to  $t_0$ . The simplest way to do this is with Euler's Method which simply approximates the average of the derivative by the value of the derivative at the beginning of the interval as follows

$$\text{Average of } \frac{dy(t)}{dt} \text{ during } t_0 - t \text{ to } t_0 \approx \frac{dy}{dt}(t_0 - t)$$

from which we obtain the following general expression for approximating  $y(t_0)$

$$y(t_o) = y(t_o - t) + t \frac{dy}{dt}(t_o - t)$$

So for our differential equation as follows

$$\dot{y} + 1000y = 1000x$$

we have

$$y(t_o) = y(t_o - t) + t \frac{dy}{dt}(t_o - t) = y(t_o - t) + t[-1000y(t_o - t) + 1000x(t_o - t)]$$

If we now make Euler's approximation at the time  $t_o = n t$  we obtain

$$y(n t) = y((n-1) t) + t[-1000y((n-1) t) + 1000x((n-1) t)]$$

Now make use of the following substitutions

$$y[n] = y(n t) \quad y[n-1] = y((n-1) t) \quad x[n-1] = x((n-1) t)$$

to obtain the difference equation for  $y[n]$  corresponding to Euler's approximation with  $t = 10^{-4}$  sec.

5. We call the result from Problem (4) as follows

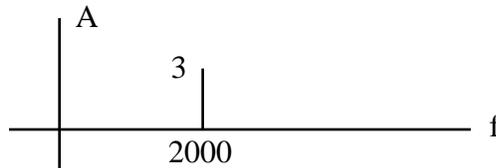
$$y[n] - 0.9y[n-1] = 0.1x[n-1]$$

a **linear difference equation with constant coefficients**. They're the equations for *dynamic discrete systems* just like differential equations are the equations of analog dynamic circuits like our RC circuit. We will be spending a large part of this course working with these equations.

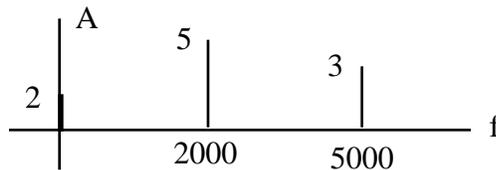
- a. What do we mean by a dynamic system
  - b. Make up your own example of a linear difference equation with constant coefficients
6. Calculate and plot the output of a discrete system with difference equation
- $$y[n] = -0.5y[n-1] + x[n] \quad \text{for } n = 0, 1, 2, 3, 4$$
- if  $x[n] = 1$  for all  $n \geq 0$  and  $y[-1] = 0$ . Put your results in a Table. Describe in words what's happening to  $y[n]$  as  $n$  increases.
7. Given all that we've done so far
- a. What do you think are the advantages of discrete systems
  - b. What do you think are the limitations of discrete systems
8. In the next two Investigations we'll be finding out how fast we have to sample a signal in order to be able to recreate it from its samples. In order to do this we need to be able to calculate and plot the spectrums of signals. We begin with sinuoids
- a. Sketch  $x(t) = 5\cos(2 \cdot 1000t)$  as a function of time
  - b. What is the amplitude  $A$  of  $x(t)$  and what is its frequency  $f$  in Hz
  - c. Describe the relationship between  $x(t) = A\cos(2 \cdot ft)$  and its *single-sided spectral plot* as follows



- d. Draw the single-sided spectral plot of  $x(t)$  in part (a)  
 e. Write out  $x(t)$  for the sinusoid with single-sided spectral plot as follows

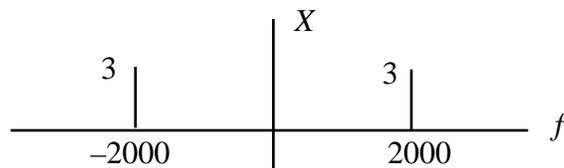


- f. Draw a single-sided spectral plot for  $x(t) = 5\cos(2 \cdot 1000t) + 3\cos(2 \cdot 2000t)$ . Hint - draw a spectral line for each sinusoid of the sum.  
 g. Write out an expression for  $x(t)$  with the following single-sided spectral plot



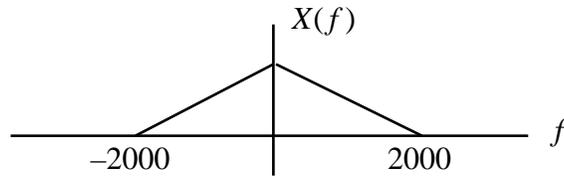
Note that a spectral line at  $f = 0$  is for a constant term.

9. The *bandwidth* of a sum of sinusoids like  $x(t) = 5\cos(2 \cdot 100t) + 3\cos(2 \cdot 200t) + \cos(2 \cdot 300t)$  is the highest frequency. Therefore the bandwidth of  $x(t)$  is  $BW = 300Hz$   
 a. What is the bandwidth of  $x(t) = 3\cos(2 \cdot 2000t) + \cos(2 \cdot 4000t)$   
 b. Write out and sketch the single-sided spectral plot of a signal  $x(t)$  with bandwidth  $BW = 7000Hz$
10. Up to now all our spectral plots have been single-sided. But double-sided spectral plots like the following



for  $x(t) = 6\cos(2 \cdot 2000t)$  are easier to work with when we calculate the spectrums of nonperiodic signals like single pulses

- a. Explain how double-sided spectral plots are different from single-sided spectral plots  
 b. Sketch the double-sided spectral plot of  $x(t) = 10\cos(2 \cdot 4000t)$
11. Again from Fourier we can show that nonperiodic signals  $x(t)$  like single pulses have spectral densities  $X(f)$  very much like our spectral plots except they're continuous like the following



We call  $X(f)$  the *Fourier Transform* of  $x(t)$ .

- a. What is the bandwidth of  $X(f)$
- b. Modify  $X(f)$  to make its bandwidth infinite - change  $X(f)$  so that it never becomes zero

12. MATLAB - Matlab is a program for doing calculations and drawing graphs that makes extensive use of matrices. Type the following code into the command window

```
>> x=5
>> x=6;
>> x
>> % x=7
>> x
```

And then make use of the results to

- a. Explain what the semicolon does
- b. Explain what the percent sign does

13. MATLAB - Type the following code into the command window.

```
>> y1 = 2*3
>> y2 = 2^3
>> y3 = sqrt (10)
>> y4 = cos (2)
>> y5 = exp (2)
>> y6 = log10 (10)
>> y7 = log (10)
```

And then make use of the results to

- a. Determine the meaning of  $\wedge$
- b. Determine the meaning of *exp*
- c. Explain the difference between  $\log_{10}(10)$  and  $\log(10)$
- d. Determine if the argument of the cosine is in radians or in degrees