ECE 306L - DIGITAL FILTERS - LAB 7
POLES, ZEROS AND FREQUENCY RESPONSE

FALL 2006 **FELZER/KANG**

OBJECTIVE

The objective of this lab is to see how the frequency response of a digital filter is related to the locations of its poles and zeros.

PRELAB

1. Given $z_1 = 2 + j3$
   a. Use your calculator to find

      $|z_1| = |2 + j3|$ = Distance in the complex plane between $2 + j3$ and the origin

   b. Use Matlab to find $|z_1| = |2 + j3|$
   c. Verify that your results in parts (a) and (b) are the same

2. Given $z_1 = 2 + j3$ and $z_2 = -1 + j2$
   a. Use your calculator to find

      $|z_1 - z_2|$ = Distance in the complex plane between z_1 and z_2

   b. Use Matlab to find $|z_1 - z_2|$
   c. Verify that your results in parts (a) and (b) are the same

3. Given $z_1 = e^{j1.2}$ and $z_2 = -1 + j2$
   a. Use your calculator to find $|z_1 - z_2|$
   b. Use Matlab to find $|z_1 - z_2|$
   c. Verify that your results in parts (a) and (b) are the same

4. Given the complex number $z = e^{jb}$
   a. Where are the points z in the complex plane. Draw a picture to illustrate
   b. What happens to z as b increases

5. Find the points $e^{j2\pi f / f_s}$ in the complex plane when
   a. $f = 0$
   b. $f = f_s/4$
   c. $f = f_s/2$

6. Given the following transfer function

$$H(z) = 2\frac{(z-1)(z+1)}{\left(z - 0.8e^{j\pi/2}\right)\left(z - 0.8e^{-j\pi/2}\right)}$$

   a. What are the poles and zeros
   b. Sketch the pole-zero diagram. Be sure to draw the unit circle

c. Why do we draw the unit circle
d. Make use of the pole-zero diagram to sketch the frequency response for $-f_s/2 \le f \le f_s/2$
e. Is this filter lowpass, highpass or bandpass. How can you tell

7. Draw the pole-zero diagram of a
   a. 1st order lowpass recursive digital filter
   b. 2nd order lowpass recursive digital filter

8. Draw the pole-zero diagram of a
   a. 1st order highpass recursive digital filter
   b. 2nd order highpass recursive digital filter

LAB

1. Write a Matlab program to plot the following distance in the complex plane

$$d(f) = \left| e^{j2\pi f/f_s} - 0.8 \right|$$

   as a function of f for $-f_s/2 \le f \le f_s/2$ for $f_s = 10^4$ samples/sec

2. Given the following first order lowpass recursive digital filter with transfer function

$$H(z) = b_0 \frac{z+1}{z-p_1}$$

   a. Write a Matlab function for plotting the frequency response $\left| H\left(e^{j2\pi f/f_s}\right) \right|$ as a function of f_s, b_0 and p_1
   b. Run your program in part (a) for $b_0 = 2$ and $p_1 = 0.8$ in the frequency range $-f_s/2 \le f \le f_s/2$ for $f_s = 5000$ samples/sec
   c. What does your graph predict for the sinusoidal steady state response to $x(t) = 5\cos(2\pi 1000t)$ sampled at $f_s = 5000$ samples/sec
   d. Use Simulink to verify your result in part (c)

3. Given the following first order highpass recursive digital filter with transfer function

$$H(z) = b_0 \frac{z-1}{z-p_1}$$

   a. Write a Matlab function for plotting the frequency response $\left| H\left(e^{j2\pi f/f_s}\right) \right|$ as a function of f_s, b_0 and p_1 for $f_s = 5000$ samples/sec
   b. Run your program in part (a) for $b_0 = 2$ and $p_1 = -0.8$
   c. What does your graph predict for $y(t)$ when $x(t) = 5\cos(2\pi 5000t)$
   d. Verify your result with Simulink

4. Run your programs in Problems (2) and (3) with values of b_0 so that
   a. The gain of the lowpass is one at $f = 0$
   b. The gain of the highpass is one at $f = f_s/2$

5. Run your programs in Problems (2) and (3) to find out what happens to

a. The frequency response of the first order lowpass as the pole moves closer to +1. Draw several graphs on the same plot
b. The frequency response of the first order highpass as the pole moves closer to -1. Draw several graphs on the same plot

6. Write a Matlab function to plot the frequency response of a 2nd order bandpass recursive digital filter as follows

$$H(z) = b_0 \frac{(z+1)(z-1)}{(z-p_1)(z-p_2)}$$

with b_0 calculated so that the maximum gain of the bandpass filter is equal to one

7. Make use of your 2nd order program to see how the locations of the poles affects the frequency response

POSTLAB

1. How does the location of the pole affect the sharpness of the frequency response of the 1st order lowpass

2. How does the location of the pole affect the sharpness of the frequency response of the 1st order highpass

3. How does the location of the poles affect the sharpness of the frequency response of the 2nd order bandpass