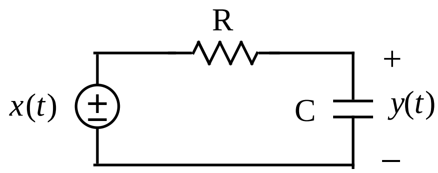
ECE 306L - FOURIER SERIES - LAB 4
HOW FILTERS AFFECT THE SPECTRUMS OF PULSE TRAINS

FALL 2006 **FELZER/KANG**

OBJECTIVE

The objective of this lab is to see how the output $y(t)$ of a lowpass filter like the following

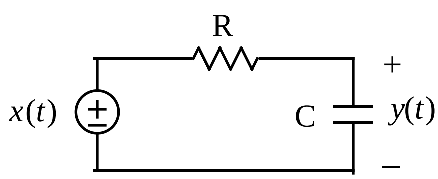


depends on both the frequency response of the filter and spectrum of the input when $x(t)$ is a pulse train

PRELAB

1. What do we mean by the frequency response of a linear circuit

2. The objective of this first problem is to find the transfer function $G(j2\pi f)$ of our first order RC circuit as follows

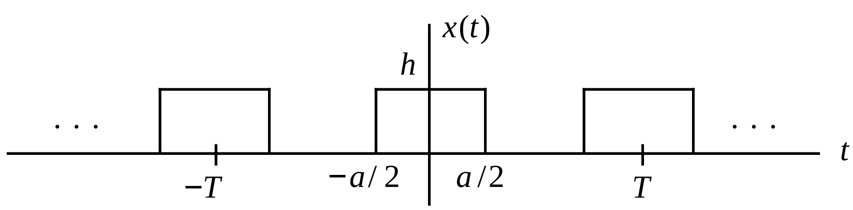


   a. Draw the phasor circuit
   b. Analyze your phasor circuit to show that its transfer function is equal to

$$G(j2\pi f) = \frac{1}{1 + j2\pi fRC}$$

3. Make use of Matlab to obtain a *full page* plot of the magnitude of the transfer function $G(j2\pi f)$ with $|G(j2\pi f)|$ plotted on a **linear scale** and f plotted on a **log scale**

4. Now suppose the input $x(t)$ of our RC circuit with $R = 10K$ is a 1/2 duty cycle pulse train as follows



   of amplitude $h = 2$ and period $T = 1\text{msec}$

   a. Find C so that the RC time constant τ is related to the pulse width a by $a = 5\tau$
   b. Sketch what you expect the circuit's response to the pulse train looks like

c. Use time domain arguments involving τ to explain your graph in part (d)
d. Use Matlab to plot the magnitude of the envelope $X_{env}(f)$ with a dashed line
e. Draw in the spectral lines by hand
f. Use Matlab to plot the magnitude of the envelope $Y_{env}(f)$ as follows with a dashed line

$$|Y_{env}(f)| = |X_{env}(f)||G(jf)|$$

g. Draw in by hand the spectral lines for $y(t)$
h. Compare the amplitudes of the spectral lines of $x(t)$ and $y(t)$
i. Make use of your result in part (h) to explain why $x(t)$ and $y(t)$ look so much alike
j. Use Matlab to obtain the magnitudes in dBV of the first ten spectral lines of the one-sided spectral plot of $y(t)$

5. Again suppose $x(t)$ is a 1/2 duty cycle pulse train of amplitude $h = 2$ and period $T = 1$ msec but this time suppose that the RC time constant τ is related the pulse width a by $a = 2\tau$
   a. Find C so that $a = 2\tau$
   b. Sketch what you expect the circuit's response to the pulse train looks like
   c. Use time domain arguments involving τ to explain your graph in part (b)
   d. Use Matlab to plot the magnitude of the envelope $Y_{env}(f)$ as follows with a dashed line

$$|Y_{env}(f)| = |X_{env}(f)||G(jf)|$$

   e. Draw in by hand the spectral lines for $y(t)$
   f. Compare the amplitudes of the spectral lines of $x(t)$ and $y(t)$
   g. Make use of your result in part (f) to explain why $x(t)$ and $y(t)$ look different
   h. Use Matlab to obtain the magnitudes in dBV of the first ten spectral lines of the one-sided spectral plot of $y(t)$

6. Now suppose the RC circuit is the same as in Problem (4) but now the pulse train is 1/4 duty cycle with $h = 2$ and period $T = 1$ msec
   a. Sketch what you expect the circuit's response to the pulse train looks like
   b. Make use of time domain arguments involving τ to explain your graph in part (a)
   c. Use Matlab to plot the magnitude of the envelope $Y_{env}(f)$ as follows with a dashed line

$$|Y_{env}(f)| = |X_{env}(f)||G(jf)|$$

   d. Draw in by hand the spectral lines for $y(t)$
   e. Compare the amplitudes of the spectral lines of $x(t)$ and $y(t)$
   f. Make use of your result in part (e) to explain why $x(t)$ and $y(t)$ look different
   g. Use Matlab to obtain the magnitudes in dBV of the first ten spectral lines of the one-sided spectral plot of $y(t)$

LAB

1. The objective these measurements is to see how a first order RC filter with $a = 5\tau$ affects a 1/2 duty cycle pulse train with amplitude $h = 2$ and period $T = 1$ msec
   a. Make a screen capture of the input pulse train $x(t)$
   b. Make a screen capture of the circuit's response to the pulse train response $y(t)$
   c. Compare $x(t)$ and $y(t)$
   d. Obtain and make a screen capture of the one-sided spectral plot of $y(t)$
   e. Use Excel to record your measured magnitudes $c_0 - c_{10}$ of the first ten harmonics in dBV

f. Use Matlab to plot the magnitudes of the spectral plot

2. The objective of these measurements is to see how a first order RC filter with $a = 2\tau$ affects a 1/2 duty cycle pulse train with amplitude $h = 2$ and period $T = 1$ msec
   a. Make a screen capture of the input pulse train $x(t)$
   b. Make a screen capture of the circuit's response to the pulse train response $y(t)$
   c. Compare $x(t)$ and $y(t)$
   d. Obtain and make a screen capture of the one-sided spectral plot of $y(t)$
   e. Use Excel to record your measured magnitudes $c_0 - c_{10}$ of the first ten harmonics in dBV
   f. Use Matlab to plot the magnitudes of the spectral plot

3. The objective of these measurements is to see how the RC circuit of Problem (1) responds to a 1/4 duty cycle pulse train with $h = 2$ and period $T = 1$msec
   a. Make a screen capture of the input pulse train $x(t)$
   b. Make a screen capture of the circuit's response to the pulse train response $y(t)$
   c. Compare $x(t)$ and $y(t)$
   d. Obtain and make a screen capture of the one-sided spectral plot of $y(t)$
   e. Use Excel to record your measured magnitudes $c_0 - c_{10}$ of the first ten harmonics in dBV
   f. Use Matlab to plot the magnitudes of the spectral plot

POSTLAB

1. Compare your calculated and measured results for the magnitudes of the spectral lines of $y(t)$ at the output of the RC filter in Lab Problem (1) when the input is a 1/2 duty cycle pulse train with $a = 5\tau$, $h = 2$ and period $T = 1$ msec

2. Compare your calculated and measured results for the magnitudes of the spectral lines of $y(t)$ at the output of the RC filter in Lab Problem (2) when the input is a 1/2 duty cycle pulse train with $a = 2\tau$, $h = 2$ and period $T = 1$ msec

3. In Lab Problem (2) we increased τ from $\tau = a/5$ to $\tau = a/2$
   a. Describe how this affected the output of the filter
   b. Use time domain arguments to explain what happened
   c. Use frequency domain arguments to explain what happened - compare spectral plots at the input and output of the filter

4. Compare your calculated and measured results for the magnitudes of the spectral lines of $y(t)$ at the output of the RC filter in Lab Problem (3) when the input is a 1/4 duty cycle pulse train with $h = 2$ and period $T = 1$ msec

5. In Lab Problem (3) we decreased the duty cycle of the pulse train from 1/2 to 1/4
   a. Describe how this affected the output of the filter
   b. Use time domain arguments to explain what happened
   c. Use frequency domain arguments to explain what happened